INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date____________FN/AN, Time: 3 Hrs., Full Marks: 120, Dept.:AG,AR,ME,MF,NA,PH
No. of Students: 239 **End Autumn Semester Examination 2011**
Sub. No.: ME21101 **Sub. Name: Fluid Mechanics**
2nd Yr. B.Tech.(H)/B.Arch.(H)/M.Sc./M.Tech.(Dual)

Instructions: Attempt all questions. Symbols have their usual meanings. Please explain your work carefully. **Clearly indicate the coordinate system used in your analysis.** Make suitable assumptions wherever necessary. Please state your assumptions clearly. **If you use the global or integral forms of the conservation laws, clearly indicate your choice of control volume using dotted lines.**

The following information may be useful:

The density of water is 1000 kg/m^3. The acceleration due to gravity is 9.8 m/s^2.

The divergence and curl of a vector field, $\mathbf{v} = \mathrm{v_r}\mathbf{e_r} + \mathrm{v_\theta}\mathbf{e_\theta} + \mathrm{v_z}\mathbf{e_z}$, in cylindrical coordinates are given by

$$\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\,\mathrm{v_r}) + \frac{1}{r}\frac{\partial \mathrm{v_\theta}}{\partial \theta} + \frac{\partial \mathrm{v_z}}{\partial z},$$

$$\nabla\times\mathbf{v} = \left(\frac{1}{r}\frac{\partial \mathrm{v_z}}{\partial \theta} - \frac{\partial \mathrm{v_\theta}}{\partial z}\right)\hat{\mathbf{e}}_r + \left(\frac{\partial \mathrm{v_r}}{\partial z} - \frac{\partial \mathrm{v_z}}{\partial r}\right)\hat{\mathbf{e}}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(r\,\mathrm{v_\theta}) - \frac{\partial \mathrm{v_r}}{\partial \theta}\right)\hat{\mathbf{e}}_z .$$

The Navier-Stokes equations for a constant-property fluid may be expressed in cylindrical coordinates as

$$\rho\left(\frac{D\mathrm{v_r}}{Dt} - \frac{\mathrm{v_\theta^2}}{r}\right) = -\frac{\partial P}{\partial r} + \mu\left(\nabla^2 \mathrm{v_r} - \frac{\mathrm{v_r}}{r^2} - \frac{2}{r^2}\frac{\partial \mathrm{v_\theta}}{\partial \theta}\right),$$

$$\rho\left(\frac{D\mathrm{v_\theta}}{Dt} + \frac{\mathrm{v_r}\,\mathrm{v_\theta}}{r}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left(\nabla^2 \mathrm{v_\theta} - \frac{\mathrm{v_\theta}}{r^2} + \frac{2}{r^2}\frac{\partial \mathrm{v_r}}{\partial \theta}\right),$$

$$\rho\frac{D\mathrm{v_z}}{Dt} = -\frac{\partial P}{\partial z} + \mu\,\nabla^2 \mathrm{v_z},$$

where $P = p - \rho\mathbf{g}\cdot\mathbf{r}$, $\mathbf{g}$ is the gravitational acceleration, $\mathbf{r} = r\,\mathbf{e_r} + z\,\mathbf{e_z}$ is the position vector,

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathrm{v_r}\frac{\partial}{\partial r} + \frac{\mathrm{v_\theta}}{r}\frac{\partial}{\partial \theta} + \mathrm{v_z}\frac{\partial}{\partial z},$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}.$$

1. A water jet pump has jet area 0.009 m^2 and jet speed V_j = 30.5 m/s. The jet is within a secondary stream of water having speed V_s = 3 m/s (see Figure 1). The total area of the duct (the sum of the jet and secondary stream areas) is 0.07 m^2.The water is thoroughly mixed in the jet pump. The velocity profile at the exit may be approximated by a uniform profile. The pressure of the jet and secondary stream are the same at the pump inlet. The drag on the walls of the duct may be neglected.

(a) Explain why Bernoulli's equation cannot be used to determine the pressure rise, $p_2 - p_1$. Here, p_1 is the pressure at the inlet and p_2 is the pressure at the exit.

(b) Determine the flow speed at the pump exit.

(c) Calculate the pressure rise, $p_2 - p_1$. **2+8+10=20**

2. Laminar to turbulent transition in two-dimensional boundary-layer flow over a flat plate takes place at Reynolds number $\mathrm{Re}_c = \rho U x_c / \mu = 10^6$, where U is the free-stream speed, ρ is the density of the fluid, μ is the dynamic viscosity of the fluid, and x_c is the critical distance from the leading edge where the

transition occurs (see Figure 2). The boundary-layer is laminar over a distance x_c of the plate, from the leading edge. Then transition takes place and the boundary layer becomes turbulent. It may be assumed that the boundary-layer is turbulent from location $x = x_c$ to $x = L$, where L is the length of the plate. The boundary-layer thickness at the location x_c is δ_c. The thickness, δ, of the laminar boundary layer at a distance x from the leading edge is described by the equation $\frac{\delta}{x} = \frac{5}{\mathrm{Re}_x^{1/2}}$. The local skin friction coefficient, $c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2}$, for a laminar boundary layer is given by $c_f = \frac{0.664}{\mathrm{Re}_x^{1/2}}$. Here $\mathrm{Re}_x = \rho U x / \mu$ is the local Reynolds number and τ_w is the wall shear stress at a distance x from the leading edge. It is known that the thickness of a turbulent boundary-layer is described by the equation $\frac{\delta}{x} = \frac{0.382}{\mathrm{Re}_x^{1/5}}$, while the local skin-friction coefficient for a turbulent boundary-layer is $c_f = \frac{0.0594}{\mathrm{Re}_x^{1/5}}$. The turbulent boundary-layer may be thought to grow from location $x = x_c - l_c$, as shown in Figure 2. Thus, the distance 'x' in the equation describing the growth of the turbulent boundary-layer and the equation describing the variation of local skin friction factor for a turbulent boundary-layer, may be interpreted as the distance along the plate measured from a virtual origin located at $x = x_c - l_c$. The length, l_c, shown in Figure 2, can be obtained from the turbulent boundary-layer growth equation. If the total length, L, of the plate is 2 m, the width of the plate is 1 m, the free-stream speed, U, is 50 m/s, the viscosity, μ, of the fluid is 1.82 x 10^{-5} kg m^{-1} s^{-1} and the density, ρ, of the fluid is 1.2 kg/m^3, calculate

(a) the transition distance x_c,

(b) the boundary-layer thickness, δ_c, at the transition point,

(c) the distance l_c if δ_c is also the turbulent boundary-layer thickness at the critical Reynolds number,

(d) the turbulent boundary-layer thickness at the end of the plate,

(e) the drag over a distance x_c of the plate from the leading edge,

(f) the drag on the plate from location $x = x_c$ to $x = L$. **4+4+3+3+3+3=20**

3. A 8 mm diameter, 3 m long (AB+BC) smooth walled tube is inserted into a 0.5 m^3 capacity water container kept on a high table, as shown in Figure 3.The water flows out through the tube. The minor loss coefficient, k_l, is 0.3 at the entrance 'A' to the tube and 2.5 for the fully open valve. The dynamic viscosity of water is 1 x 10^{-3} Pa.s.

(a) Write down the equation for pressure difference over the full length of the tube if the flow rate is Q m^3/s and the flow is assumed to be laminar.

(b) If the water level in the container is 0.5 m when full, find the time required to fill a 250 cm^3 bottle kept under the outlet of the tube.

(c) What is the gauge pressure at the highest point 'B' when the tank is full ?

(d) Find the approximate time required to empty the tank. You may assume the flow rate to vary linearly with water head. **5+5+5+5=20**

4. A liquid of density ρ and viscosity μ flows by gravity through a hole of diameter d at the bottom of a tank of diameter D (see Figure 4). It takes time t_{empty} to drain out the liquid of height h in the tank. So, we may say that $t_{empty} = f(\rho, \mu, g, d, D, h)$. Here, g is the gravitational acceleration. The designers want to experimentally predict the time required to drain out ethylene glycol from the tank. As ethylene

glycol is expensive, they want to do the experiment using water as the test liquid in a one-fourth scale geometrically similar model of the prototype tank. The kinematic viscosity, ν, of water is $\nu = \mu / \rho = A\exp(B/T)$, where $A = 1.06 \times 10^{-9}$ m^2/s, $B = 2022$ K and T is the temperature.

(a) Find the dimensionless parameters that relate t_{empty} to the other variables.

(b) The temperature of the ethylene glycol in the prototype tank is 60^{0}C, at which the kinematic viscosity is $\nu = 4.75 \times 10^{-6}$ m^2/s. At what temperature should the water in the model experiment be set in order to ensure complete similarity between model and prototype ?

(c) It takes 4.53 min to drain the model tank. Predict how long it will take to drain the ethylene glycol from the prototype tank. **10+5+5 = 20**

5. Consider steady laminar axisymmetric fully developed incompressible flow of a constant-property Newtonian fluid through a horizontal concentric cylindrical annulus of inner radius R_1 and outer radius R_2, driven by a constant externally applied axial pressure gradient.

(a) Solve the Navier-Stokes system of equations to determine the radial distribution of the axial velocity, $V_z(r)$.

(b) Determine the average velocity, $V_{z_{av}} = Q/A$, through the annulus, where Q is the volume flow rate and A is the cross-sectional area of the annular duct.

(c) Use your answer to part (b) to obtain an expression for the Darcy friction factor, f, defined by the equation, $H_{loss} = f \dfrac{L}{D_h} \dfrac{(V_{z_{av}})^2}{2g}$, where H_{loss} is the head loss over a length L of the duct, g is the magnitude of the gravitational acceleration and D_h is the hydraulic diameter. **10+4+6=20**

6 (a) Consider cross flow past a long stationary circular cylinder of radius a. Far upstream of the cylinder, the flow is uniform, with a velocity field $\mathbf{v} = U_\infty \mathbf{i}$, where U_∞ is the speed of the uniform stream approaching the cylinder. Two-dimensional steady incompressible irrotational flow over the cylinder is represented by the velocity field, $\mathbf{v} = U_\infty \left[1 - \left(\dfrac{a}{r}\right)^2\right] \cos\theta \, \mathbf{e_r} - U_\infty \left[1 + \left(\dfrac{a}{r}\right)^2\right] \sin\theta \, \mathbf{e}_\theta$, in cylindrical polar coordinates. Here, the origin of the coordinate system is at the centre of the cylinder, $\mathbf{e_r}$ is the unit vector in the radial direction, $\mathbf{e}_\theta$ is the unit vector in the azimuthal direction, r is the radial coordinate and θ is the angle measured from the rear stagnation point. Obtain an expression for the pressure coefficient $C_p = \dfrac{p_s - p_\infty}{\frac{1}{2}\rho U_\infty^2}$, where p_s is the pressure on the surface of the cylinder, p_∞ is the pressure far upstream of the cylinder and ρ is the density of the fluid.

(b) An Arctic hut in the shape of a semi-circular cylinder has a radius a. A wind of velocity U_∞ batters the hut and threatens to raise it off its foundations due to the lift of the wind. The lift is partly due to the fact that the entrance to the hut is at ground level at the location of the stagnation pressure. A clever occupant sized up the situation quickly and re-located the entrance at an angle θ_0 from the ground level, which caused the net force on the hut to vanish (see Figure 5). What is the angle θ_0? The flow over the hut may be represented by the velocity field for flow past a cylinder, given in part (a) above. Assume that the opening is very small compared to the radius a. Note that the static pressure, p_i, inside the hut depends on the angle θ_0. **7+13 = 20**

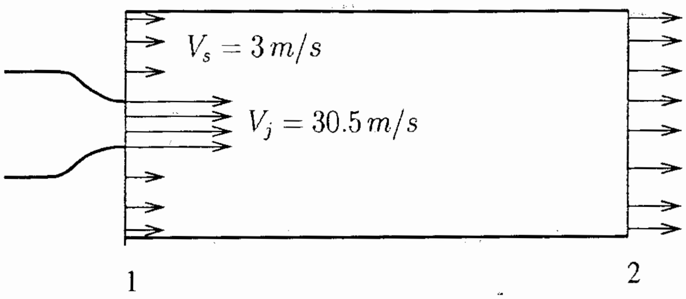

Figure 1 (for Question 1)

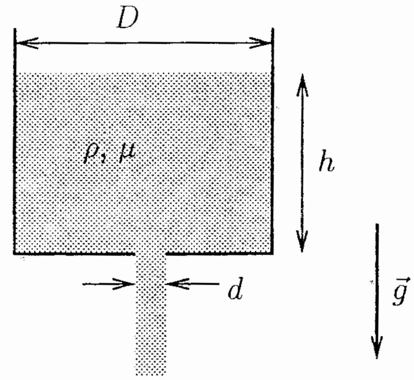

Figure 4 (for Question 4)

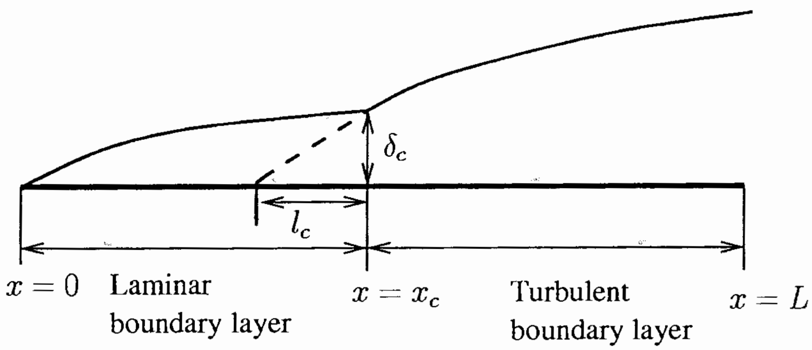

Figure 2 (for Question 2)

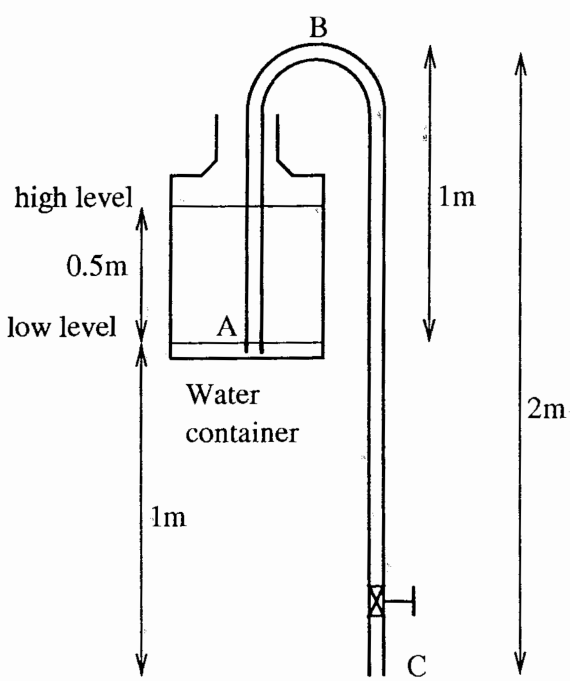

Figure 3 (for Question 3)

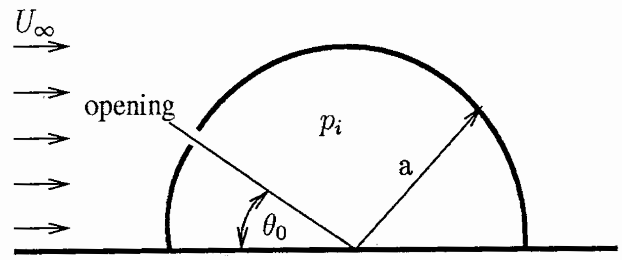

Figure 5 (for Question 6)