INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date 03-10-2012 FN/AN, Time: 2 Hrs., Full Marks: 70, Dept.: AE, AG, CH, ME, NA
No. of Students: 256 Mid Autumn Semester Examination 2012
Sub. No.: ME21101 Sub. Name: Fluid Mechanics
2nd Yr. B.Tech.(H)/M.Tech.(Dual)

Instructions: Attempt all questions. Symbols have their usual meanings. Please explain your work carefully. **Clearly indicate the coordinate system used in your analysis.** Make suitable assumptions wherever necessary. Please state your assumptions clearly. The following information may be useful: The density of water is 1000 kg/m^3. The acceleration due to gravity is 9.8 m/s^2.

1. A cylindrical glass beaker is partially filled with two immiscible liquids, a layer of water below and a layer of oil on top. The density, ρ_2, of water is greater than the density, ρ_1, of the oil. The axis of the cylindrical beaker is vertical. The top of the beaker is open to the atmosphere. The beaker and its contents are rotated about the vertical axis at a constant angular speed, Ω, and the liquids eventually rotate as rigid bodies. The free surface of the oil (that is, the oil-air interface) and the oil-water interface are observed to reach a curved shape, as shown in figure 1, the respective layers of oil and water having the depths H_1 and H_2 at the centerline of the cylindrical beaker. Surface tension effects may be neglected.

(a) Derive an expression for the pressure, $p_1(r,z)$, in the oil at a height z above the base of the beaker and a radius r from the centerline, using as parameters only the known quantities H_1, H_2, g, Ω, p_{atm} and ρ_1. Here, g is the gravitational acceleration and p_{atm} is the atmospheric pressure.

(b) Using your answer to part (a), determine the shape of the free surface of the oil. Express the equation of the free surface in the form $z = f_1(r; H_1, H_2, g, \Omega)$, where the semi-colon separates the constant parameters H_1, H_2, g and Ω from the independent variable r.

(c) Determine the pressure at the oil-water interface on the centerline.

(d) As in part (a), derive an expression for the pressure, $p_2(r,z)$, in the water by using your answer to part (c), using as parameters only the known quantities H_1, H_2, g, Ω, p_{atm}, ρ_1 and ρ_2.

(e) Determine the shape of the oil-water interface. Express the equation of the oil-water interface in the form $z = f_2(r; H_2, g, \Omega)$, where the semi-colon separates the constant parameters H_2, g and Ω from the independent variable r. **[4+2+1+3+5 = 15 marks]**

2. Consider steady two-dimensional incompressible boundary-layer flow of a constant-property fluid over a large horizontal flat plate of length L and width b, aligned with the direction of a uniform oncoming free stream, of velocity $\mathbf{v} = u_\infty \mathbf{i}$ (see figure 2). The velocity distribution at the trailing edge, $x = L$, of the plate may be approximated by $u = u_\infty \left[\frac{3}{2}\left(\frac{y}{\delta_L}\right) - \frac{1}{2}\left(\frac{y}{\delta_L}\right)^3 \right]$ for $0 \le y \le \delta_L$, and $u = u_\infty$ for $y \ge \delta_L$, where δ_L is the thickness of the boundary-layer at $x = L$. Here, x is a coordinate along the plate, u is the component of velocity in the x-direction, y is the coordinate normal to the plate (measured from the plate) and the origin is at the leading edge of the plate. The pressure may be assumed to be uniform. Use integral conservation of mass and momentum to determine the drag force on one side of the plate. Indicate your choice of control volume using dotted lines. Express your answer in terms of the parameters u_∞, δ_L, b and the density, ρ, of the fluid. **[15 marks]**

3. A viscous fluid (specific weight, ρg = 1.26 kN/m^3; viscosity = 1.4 N.s/m^2) is contained between two infinite parallel plates, as shown in figure 3. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube

manometer connected between two points along the bottom indicates a differential reading of 0.25 cm. The upper plate moves with a velocity of 6 x 10^{-3} m/s. The distance between the two plates is 2.5 cm. The specific weight of the manometer fluid is 15.7 kN/m^3 and the distance between the two pressure taps on the lower plate is 15 cm. Assume steady, laminar flow.

a) Write down the continuity equation and the complete three Navier-Stokes equations.
b) Reduce the three Navier-Stokes equations to the maximum possible, giving justifications at each step.
c) Solve the equations of part (b) to determine the velocity distribution.
d) Find out the pressure gradient as shown in the manometer.
e) At what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? **[4+4+4+3+5 = 20 marks]**

4. Water of flow rate 10kg/s is accelerated by a nozzle to a speed of 20m/s, and strikes a stationary horizontal channel placed at an angle θ to the jet direction as shown in figure 4. After the strike the water splatters off in two directions horizontally as shown in the figure. Flow is frictionless.

(a) Draw the control volume required for the analysis, clearly showing the inlet and outlets.
(b) What is the force in the x-direction, exerted by the water on the plate, if the flow is assumed to be inviscid ?
(c) Determine the force in the y-direction required to stop the channel from moving, when $\theta = 90^0$.
(d) Determine the distribution of water in the two streams when $\theta = 90^0$.
(e) Recalculate (c) and (d) when the angle is $\theta = 45^0$. **[2+2+4+4+8 = 20 marks]**

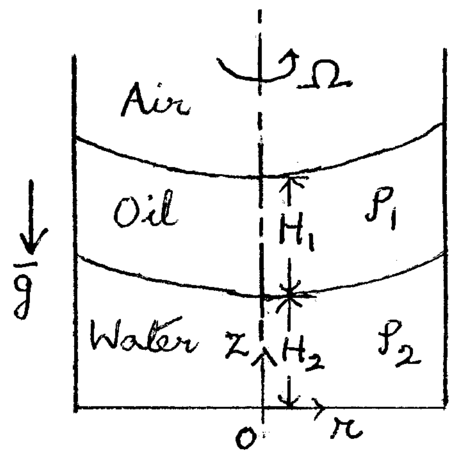

Figure 1 (for Q1)

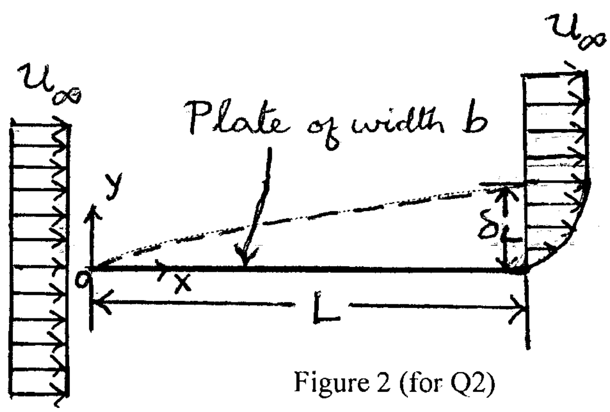

Figure 2 (for Q2)

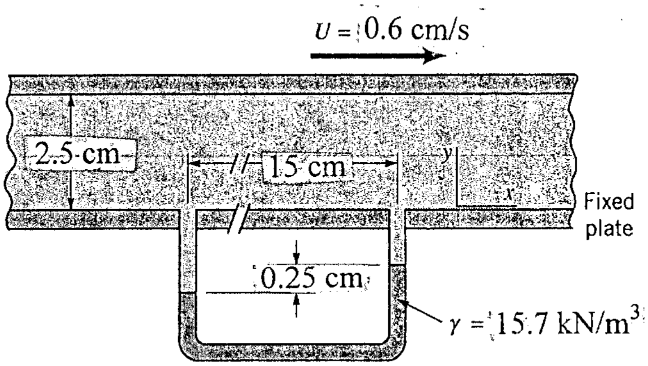

Figure 3 (for Q3)

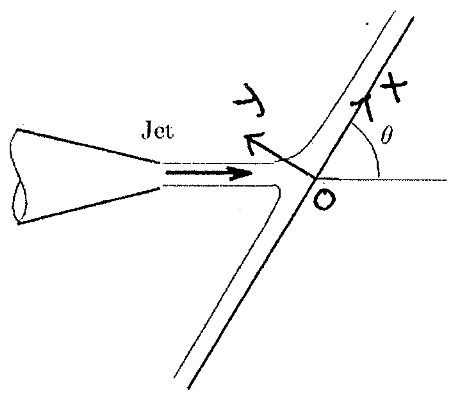

Figure 4 (for Q4)