Advanced Fluid Mechanics (ME61003)/ Fluid Mechanics (ME60011), Mid Semester Examination, September 2017, IIT Kharagpur, Full Marks = 60, Time: 2 hours

All questions are compulsory. Each Question Carries Equal Marks

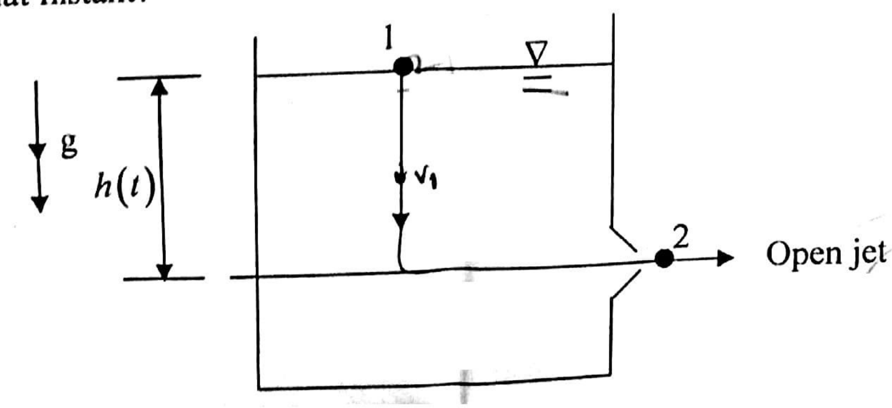
Q1. A flow field is defined by the following velocity components, where a and b are dimensional constants:

$$u = 0,\; v = a(x - z),\; w = by$$

(a) Is the flow incompressible? Justify.
(b) Obtain an expression for the stream function, if it exists.
(c) Obtain an expression for the velocity potential if it exists, for (i) $a = b$, (ii) $a = -b$.
(d) Sketch the deformed configuration of a rectangular fluid element that was having its edges originally parallel to the y and z axes, respectively, for the cases (i) $a = b$, (ii) $a = -b$.

[12 marks]

Q2. Water comes out of a tank through a nozzle, as an open jet. As a result, the level of water in the tank continuously falls. A streamline in the tank conceptually identifies in the figure below (Spanning from point 1 to point 2), the curvilinear length of which (spanning from the point 1 to the point 2) is approximately kh, where $k = 1.5$. For mathematical analysis, following assumptions can be made: (i) velocity of flow along the streamline is approximately V_1, and (ii) viscous effects are negligible. The ratio of area of cross section of the tank to that of the nozzle is 2: 1. At a given instant of time, $h = 5$ m and $V_2 = 1$ m/s. What is the local component of acceleration of flow at the point 2, at that instant?



Q3. The velocity components in an inviscid, constant density (= 1000 kg/ m^3), steady flow field are given as follows: $u = \frac{A}{2}(x + y + z)$, $v = \frac{A}{2}(x + y + z)$, $w = -A(x + y + z)$, where A is a dimensional constant, with a numerical value of 1 unit. Consider a directed line segment in the flow field, connecting the points P_1 (0, 0, 0) and P_2 (-3, 3, 0). The pressure is given as zero gauge at the origin.

(i) Is the line P_1P_2 a streamline? Justify with calculations.
(ii) Can the Bernoulli's equation be applied to find the change in pressure experienced on moving from the point P_1 to the point P_2 along the direction P_1P_2? Justify with calculations.
(iii) What is the pressure at P_2 (derive from first principles, starting from Euler's equation of motion in differential form)?

You may use the following: (A) Euler's equation of motion in rectangular coordinates is given as $-\frac{\vec{\nabla}p}{\rho}+\vec{b}=(\vec{V}\bullet\vec{\nabla})\vec{V}$, where $\vec{b}$ is the body force per unit mass. (B) The following vector identity is applicable: $(\vec{A}\bullet\vec{\nabla})\vec{A}=\frac{1}{2}\vec{\nabla}(\vec{A}\bullet\vec{A})-\vec{A}\times(\vec{\nabla}\times\vec{A})$.

Q4.

(a) Apply Reynolds transport theorem for deriving an integral expression for linear momentum conservation.

(b) Express the above equation with respect to a reference frame xyz that translates and rotates arbitrarily with respect to the inertial reference frame XYZ. Physically, this pertains to the consideration of an arbitrarily moving control volume (with which the xyz reference frame is attached) instead of a stationary control volume (with which XYZ reference frame is attached). You may consider the following relation between the acceleration relative to XYZ and that relative to xyz:

$\vec{a}_{XYZ}=\vec{a}_{xyz}+\ddot{\vec{r}}_{xyz}+2\vec{\omega}\times\dot{\vec{r}}_{xyz}+\dot{\vec{\omega}}\times\vec{r}_{xyz}+\vec{\omega}\times(\vec{\omega}\times\vec{r}_{xyz})$, where $\vec{\omega}$ is the angular velocity of the xyz reference frame, $\vec{r}_{xyz}$ is the position vector relative to that reference frame, and 'dot' denotes differentiation with respect to time.

(c) From the above, derive a differential equation for momentum conservation of an inviscid flow with respect to an arbitrarily moving reference frame, in non-conservative form.