All questions are compulsory. Each Question Carries Equal Marks

Q1. (a) Starting from the Reynolds transport theorem, derive a governing differential equation for momentum conservation for inviscid and incompressible flows. Is the same equation valid for compressible flows as well?

(b) Consider the following constitutive form for stresses in a Newtonian fluid:

$$\tau_{ij} = -p\delta_{ij} + \lambda(\nabla \bullet \vec{u})\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \text{ where } \delta_{ij} = 1 \text{ if } i = j \text{ and is zero otherwise.}$$

Here p is the thermodynamic pressure and $\vec{u}$ is the velocity vector.

(i) Derive an expression for the relationship between μ and λ for dilute, monoatomic gases (assume that for such substances the mechanical and thermodynamic pressures are identical). Is this derived expression restricted to any approximation?

(ii) Physically interpret the algebraic sign of λ, considering the above case.

[15 + 10 = 25 Marks]

Q2.(a)

(i) Physically explain the mechanism of stretching of a vortex in a turbulent flow (in not more than 2 sentences).

(ii) Can the vortex stretching turn out to be an important phenomenon over the small eddy length scales? Justify (in not more than 2 sentences).

(iii) Physically explain whether spatial average and ensemble averages may be same for stationery turbulence (in not more than 2 sentences).

(b) Show that the ratio of the largest eddy length scale and the Kolmogorov length scale is of the order of $Re^{3/4}$ where Re is based on the largest eddy scale.

(c) Starting from the Navier Stokes equation, derive the vorticity transport equation for incompressible flows. Hence, show that there is no vortex stretching in 2-D flows. Is the same conclusion valid for turbulent flows?

[9 + 6 + 10 = 25 Marks]

Q3.

(a) A continuous belt, passing upward through a chemical bath at speed U_0 (see the figure in the next page), picks up a liquid film of thickness h, density ρ, and viscosity. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assume that the flow is fully developed and laminar with zero pressure gradient, and the atmosphere produces no shear stress at the outer surface of the film. State clearly the boundary conditions to be satisfied by the velocity at y=0 and y=h. Obtain an expression for the velocity profile.

(b) Consider an infinite flat plate that executes oscillations in contact with a Newtonian fluid, with a velocity given by: $u(0,t) = U_0 \sin \omega t$. Derive an expression for the incipient velocity field and represent it schematically.
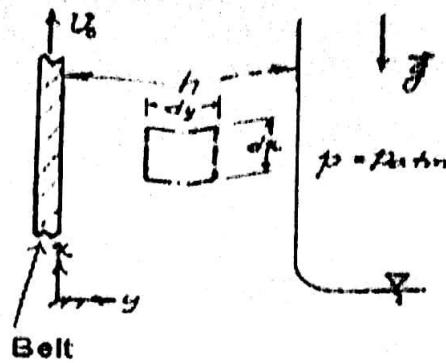
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[15 + 10 = 25 Marks]

Fig. Q3(a)

Q4.

(a) The free stream velocity for a flow is expressed in terms of the stream-wise boundary layer coordinate as $U_\infty = cx^m$, where m is a constant. Write the boundary layer equations for this case in the most simplified form. There is no need to solve the equations.

(b) Assuming $\dfrac{u}{U_\infty} = \dfrac{dF}{d\eta}$, where η is an appropriate similarity variable for boundary layer over a flat plate, derive a governing ordinary differential equation for F. Also mention the appropriate boundary conditions, clearly indicating how those are derived. Is it possible to consider $F = 1$ at the solid boundary, coupled with the above equation? Justify with reasons. There is no need to solve the equation.

(c) A flat plate is exposed to a fluid flow with a free stream parallel to the axis of the plate. In another case, this plate is replaced by another plate of half the length of the previous plate, all other conditions remaining unaltered. In both the cases, flow over the entire length of the plate is laminar. With an order of magnitude analysis, determine the ratio of the drag coefficients for these two cases.

[5 + 12 + 8 = 25 Marks]

APPENDIX: Useful Vector Identities

$$(A \cdot \nabla)A = \frac{1}{2}\nabla(A \cdot A) - A \times (\nabla \times A)$$

$$\nabla \times (A \times B) = A(\nabla \cdot B) - B(\nabla \cdot A) + (B \cdot \nabla)A - (A \cdot \nabla)B$$

$$\nabla \times (\nabla f) = 0$$

$$\nabla \cdot (\nabla \times A) = 0$$