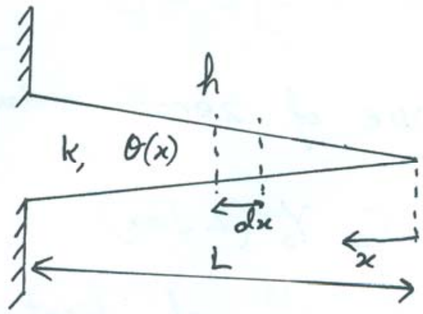
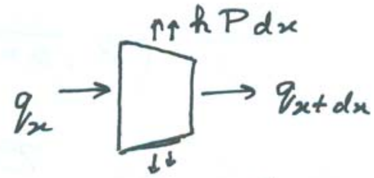
ME - 30005 - Mid Sem

Perimeter $= P(x)$

1) Cross Sectional Area $= A_c(x)$



$A_c(x) = W \frac{t}{L} x$; $P(x) = 2\left[W + \frac{t}{L}x\right]$



Let us consider an element of length 'dx' along the fin. Energy balance at steady state will give

$$\frac{d}{dx}\left[k A_c(x) \frac{d\theta}{dx}\right] - h P(x)\theta = 0 \qquad \text{where } \theta = T(x) - T_\infty$$

$$\text{or} \quad \frac{d}{dx}\left(A_c(x) \frac{d\theta}{dx}\right) - \frac{h P(x)}{k}\theta = 0$$

$$\cancel{\text{or} \quad A_c(x) \frac{d^2\theta}{dx^2} + \frac{dA_c}{dx}}$$

$$\text{or} \quad \frac{d}{dx}\left[\frac{Wt}{L} x \frac{d\theta}{dx}\right] - \frac{h}{k}\left[2W + \frac{t}{L}x\right]\theta = 0$$

Assuming $W \gg t$ (reasonable for 1-D conduction),

$$\frac{d}{dx}\left[\frac{Wt}{L} x \frac{d\theta}{dx}\right] - \frac{2hW}{k}\theta = 0$$

$$\text{or} \quad \frac{d}{dx}\left(x \frac{d\theta}{dx}\right) - \frac{2hL}{kt}\theta = 0$$

$$\text{or} \quad x \frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - \frac{2hL}{kt}\theta = 0$$

$$\text{or} \quad x \frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - a\theta = 0$$

where $\boxed{a = \frac{2hL}{kt}}$

The above equation can be re-written as

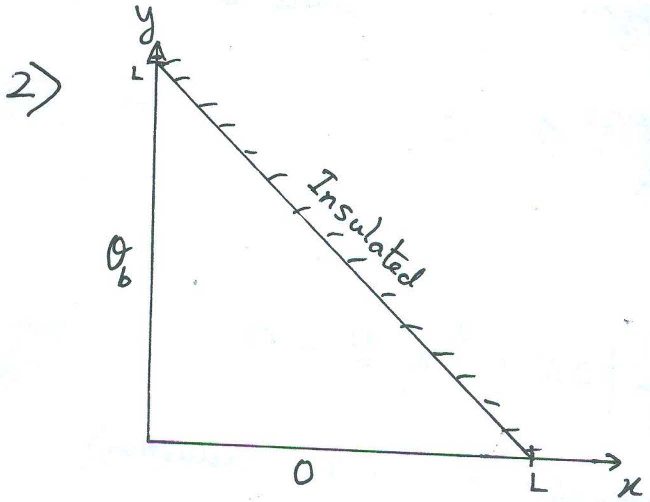
$$(\sqrt{x})^2 \frac{d^2\theta}{dx^2} + \sqrt{x}\,\frac{d\theta}{dx} - \left[(\sqrt{ax})^2 + 0\right]\theta = 0$$

which is Bessel's ODE of zeroth order

$$\Rightarrow \quad \theta(x) = C_1 J_0\left(2\sqrt{ax}\right) + C_2 Y_0\left(2\sqrt{ax}\right)$$

where J_0 & Y_0 are Bessel functions of first and second kind and of zeroth order

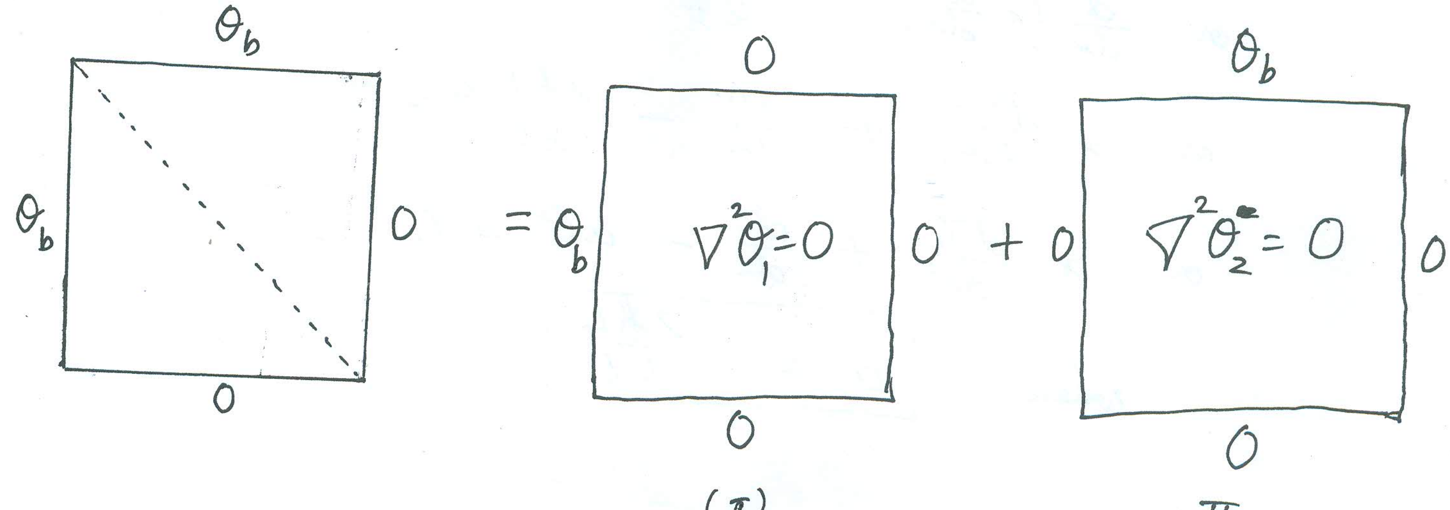
2)



This problem is that of 2-D conduction with ~~sys~~ insulated diagonal / hypotenuse. One can ~~impose~~ ~~symmetr~~ leverage symmetry condition

i) Leverage symmetry

ii) Method of superposition



(I) II

$$\underline{\theta(x,y) = \theta_1(x,y) + \theta_2(x,y)}$$

(I) $\nabla^2\theta_1 = 0$ $\qquad \theta_1(0,y) = \theta_b \qquad \theta_1(x,0) = 0$

$\qquad \theta_1(L,y) = 0 \qquad \theta_1(x,L) = 0$

This can be solved using separation of variables with y as the direction of homogenity.

(II) $\nabla^2\theta_2 = 0$ $\qquad \theta_2(0,y) = 0 \qquad \theta_2(x,0) = 0$

$\qquad \theta_2(L,y) = 0 \qquad \theta_2(x,L) = \theta_b$

This can be solved using separation of variables with x as the direction of homogenity.

3) Given:

T_∞ = ambient temp. = $20^\circ C$

$T_0(\tau)$ = temp. of body surface at time $\tau = 25^\circ C$

T_i = initial temp. (of live body) = $37^\circ C$

h = heat transfer coefficient = $4\ W/m^2\text{-}K$

D_0 = diameter of body = 20-cm

L = length of body = 1.7 m $(L \gg d_0)$

k = thermal conductivity of human body = $0.8\ W/m\text{-}K$

α = thermal diffusivity of human body = $5 \times 10^{-7}\ m^2/s$

$$\text{Biot no} = Bi = \frac{h(D_0/2)}{k} = \frac{4 \times 0.1}{0.8} = 0.5 > 0.1$$

Since $Bi > 0.1$, we shall <u>not</u> use lumped capacitance method.

Instead, we shall use Heisler Charts for an infinite cylinder since $L \gg D_0$

$$\frac{T_0(\tau) - T_\infty}{T_i - T_\infty} = \frac{25-20}{37-20} = 0.3$$

From Heisler Charts (using $1/Bi = 2$)

$$F_0 = 1.5 = \frac{\alpha \tau}{R_0^2}$$

$$\Rightarrow \quad \tau = \frac{F_0 R_0^2}{\alpha} = \frac{1.5 \times (0.1)^2}{5 \times 10^{-7}} \, s$$

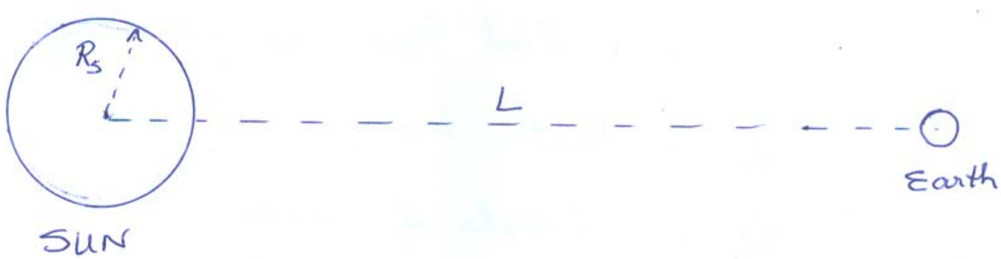
$$= 3 \times 10^4 \, s$$

$$= 8.33 \text{ hrs} \quad \text{or} \quad 8 \text{ hrs } 20 \text{ mins}$$

∴ time of death is estimated to be (17:00 − 8:20) hrs

or <u>08:40 hrs</u> or <u>8:40 a.m.</u>

4)



Temperature of sun $= T_s = 5779\,K$

a) λ_m = wavelength corresponding to max. monochromatic emissive power $e_{b\lambda}\big|_{max}$

$$= \frac{0.0029}{5779} \quad \text{using Wien's Law}$$

or $\boxed{\lambda_m = 0.5018 \, \mu\text{-}m}$

$$\therefore \quad e_{b\lambda}\Big|_{max} = \frac{2\pi C_1}{\lambda_m^5\left[e^{C_2/\lambda_m T} - 1\right]}$$ using Planck's Law

$C_1 = 5.96 \times 10^{-17}$ W-m
$C_2 = 0.0014387$ m-K

$$= 8.303 \times 10^7 \ W/m^2\text{-}\mu m$$

or $$\boxed{e_{b\lambda}\Big|_{max} = 8.303 \times 10^{13} \ W/m^3}$$

b) Emissive Power of Sun's surface $= e_b = \sigma T_s^4$ using Stefan - Boltzmann Law

$$\Rightarrow \quad e_b = 5.667 \times 10^{-8} \times (5779)^4 \ W/m^2$$

or $$\boxed{e_b = 6.32 \times 10^7 \ W/m^2}$$

$\varepsilon = 1.0$ since sun's surface is black

c) The heat flux away from the surface must vary with $1/R^2$ since area of a sphere is $4\pi R^2$. Therefore at earth's surface

$$e_{b\lambda}\Big|_{earth} = e_b\Big|_{sun} \times \frac{R_s^2}{L^2} = 8.303 \times 10^{7} \times \left(\frac{6.95 \times 10^5}{1.5 \times 10^8}\right)^2 \ W/m^2\text{-}\mu m$$

$$= \underline{\underline{1792 \ W/m^2\text{-}\mu m}}$$

$$e_b\Big|_{earth} = e_b\Big|_{sun} \times \frac{R_s^2}{L^2} = 6.32 \times 10^7 \left(\frac{6.95 \times 10^5}{1.5 \times 10^8}\right)^2 W/m^2$$

$$= \underline{\underline{1365 \ W/m^2}}$$

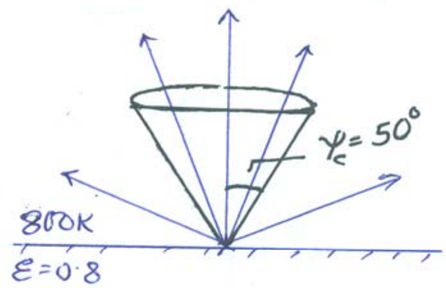
5) a) From Stefan - Boltzmann law

$$e = \varepsilon \sigma T_s^4$$

$$= 0.8 \times 5.67 \times 10^{-8} \times (800)^4 \quad W/m^2$$

or $e = 1.857 \times 10^{4} \; W/m^2$

$$\therefore \quad \boxed{i_n = e/\pi = 5908 \; W/m^2\text{-str}}$$



Given $T_s = 800K$
$\varepsilon = 0.8$
$\psi_c = 50^\circ$

To find: (a) i_n
(b) radiant flux for $\psi_c = 50°C$

Radiant Flux in a cone with $\psi_c = 50^\circ$ will be given by

$$e_{\psi_c} = \int_0^{2\pi} \int_0^{\psi_c} i_n \cos\psi \sin\psi \, d\psi \, d\phi$$

$$= 2\pi i_n \int_0^{\psi_c} \frac{\sin 2\psi}{2} \, d\psi$$

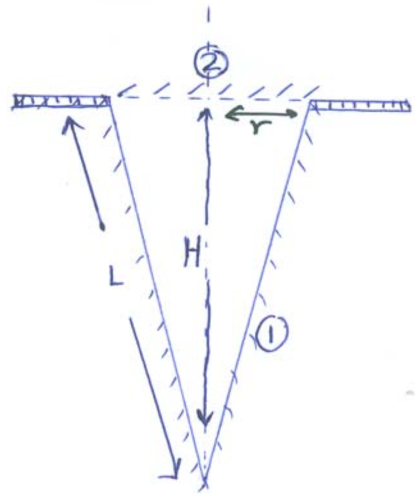
$$= \pi i_n \left[\frac{\cos 2\psi}{2} \right]_{\psi_c}^{0}$$

$$= \frac{\pi i_n}{2} \left[1 - \cos 100^\circ \right]$$

$$= \frac{\pi i_n}{2} \left(1 + 0.1736 \right)$$

or $\boxed{e_{\psi_c} = 10897 \; W/m^2}$

5 b) Close the lid of the conical cavity.



Let A_1 = inner surface of conical cavity

A_2 = inner surface of lid

$F_{2-2} = 0$ [$\because$ flat surface]

To find: F_{1-1}

$\Rightarrow F_{2-1} = 1$

Now, $F_{1-1} + F_{1-2} = 1$ → enclosure

$$\Rightarrow F_{1-1} = 1 - F_{1-2}$$

$$= 1 - \frac{A_2}{A_1} F_{2-1}$$ → reciprocal relation

$$= 1 - \frac{A_2}{A_1} \quad [\because F_{2-1} = 1]$$

$$= 1 - \frac{\pi r^2}{\pi r L}$$ where r = radius of cone $= \sqrt{L^2 - H^2}$

$$= 1 - r/L$$

or $$\boxed{F_{1-1} = 1 - \sqrt{1 - \left(\frac{H}{L}\right)^2}}$$