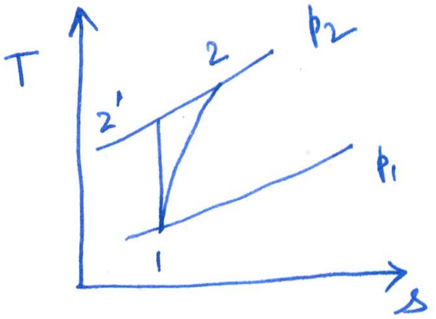
Ex

Air at 100 kPa and 300 K (static pressure and temperature) is compressed through a static pressure ratio of 2.1 in a compressor stage. The inlet and outlet velocities are 20 m/s and 125 m/s respectively. If the total-to-total efficiency is 88%, calculate the total-to-static, static-to-total and static-to-static efficiencies. ~~Plot~~ Plot them in T-s plane.

79.4 88.3 79.6



$p_1 = 100\,\text{kPa}, \quad T_1 = 300\,\text{K}.$

$$\frac{p_2}{p_1} = 2.1 \qquad \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

29/8/17

<u>Ex</u>

In a gas turbine, combustion gases flow at a rate of 1.4 kg/s. At the inlet, the static pressure and temperature are 600 kPa and 1000K ~~res~~ respectively and the velocity is 185 m/s. At the stage outlet, the static pressure is 275 kPa and the velocity is 90 m/s. If the power ~~outt~~ output of the stage is 260 kW, find the efficiency of the stage when,

(a) the stage is the first one

(b) the stage is one of the middle stages

(c) the stage is the last one.

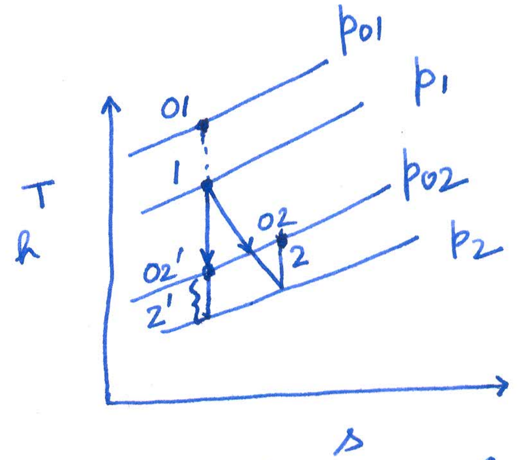
$p_1 = 600 kPa$

$T_1 = 1000 K$

$V_1 = \cancel{275\ m/s}\ 185\ m/s$

$p_2 = 275 kPa$

$V_2 = 90 m/s$



$T_{02} = 832.2 K$

SFEE $\quad -w = \left(h + \frac{V^2}{2}\right)_2 - \left(h + \frac{V^2}{2}\right)_1$

$w = h_{01} - h_{02}$

(b) $(\eta_t)_{t-t} = \dfrac{h_{01} - h_{02}}{h_{01} - h_{02}'}$

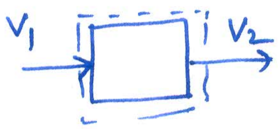
(a)

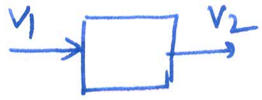
$= 86.8\%$

$T_{02}' = 804 K$

$T_{01} = 1017 K$

$T_{02} = 832.2 K$





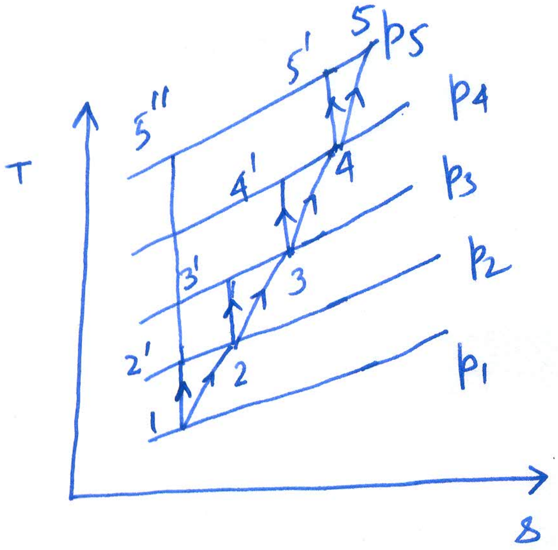
(c) $(\eta_t)_{t-s} = \dfrac{h_{01} - h_{02}}{h_{01} - h_2'}$

$T_2' = 800 K$

$= 85.2\%$

29/8/17

3



η_c ——— T_2

$$\eta_s = \frac{T_2' - T_1}{T_2 - T_1}$$

$$w_{s_1} = c_p(T_2 - T_1)$$
$$= \frac{c_p}{\eta_s}(T_2' - T_1)$$
$$= \frac{c_p T_1}{\eta_s}\left(\frac{T_2'}{T_1} - 1\right)$$
$$= \frac{c_p T_1}{\eta_s}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$w_{s_2} = \frac{c_p}{\eta_s}(T_3' - T_2)$$

$$w_{s_3} = \frac{c_p}{\eta_s}(T_4' - T_3)$$

$$w_{s_4} = \frac{c_p}{\eta_s}(T_5' - T_4)$$

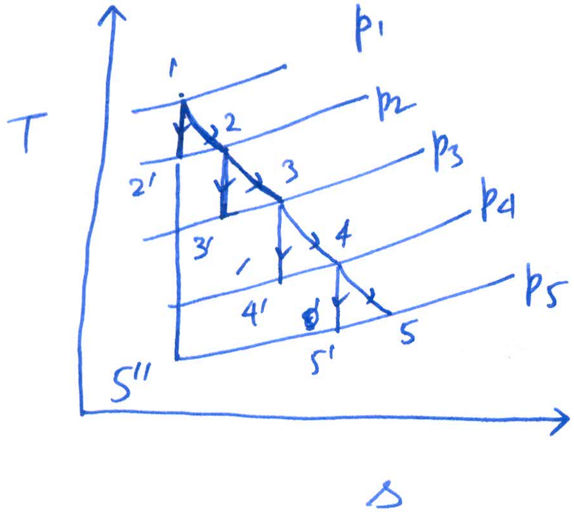
$$w = \sum w_s = \frac{c_p}{\eta_s}\left[(T_2' - T_1) + (T_3' - T_2) + (T_4' - T_3) + (T_5' - T_4)\right]$$

$$\eta_c = \frac{T_5'' - T_1}{T_5 - T_1}$$

$$c_p(T_5 - T_1) = w = \frac{c_p(T_5'' - T_1)}{\eta_c}$$

$$\frac{c_p(T_5'' - T_1)}{\eta_c} = \frac{c_p}{\eta_s}\left[(\ ) + (\ ) + (\ ) + (\ )\right]$$

$$\frac{\eta_s}{\eta_c} = \frac{[(\ ) + (\ ) + \cdots]}{(T_5'' - T_1)} > 1 \qquad \boxed{\eta_s > \eta_c}$$



$\eta_t > \eta_s$

RH = Actual dro

$$\eta_s = \frac{h_1 - h_2}{h_1 - h_2'} = \frac{T_1 - T_2}{T_1 - T_2'}$$

$$w_{s_1} = c_p(T_1 - T_2) = \eta_s(T_1 - T_2')$$

$$w_{s_2} = c_p(T_2 - T_3) = \eta_s(T_2 - T_3')$$

$$w_{s_3} = c_p(T_3 - T_4) = \eta_s(T_3 - T_4')$$

$$w_{s_4} = c_p(T_4 - T_5) = \eta_s(T_4 - T_5')$$

$$w = \sum w_s = c_p\left[(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_4 - T_5)\right]$$

$$= \eta_s\left[(T_1 - T_2') + (T_2 - T_3') + (T_3 - T_4') + (T_4 - T_5')\right]$$

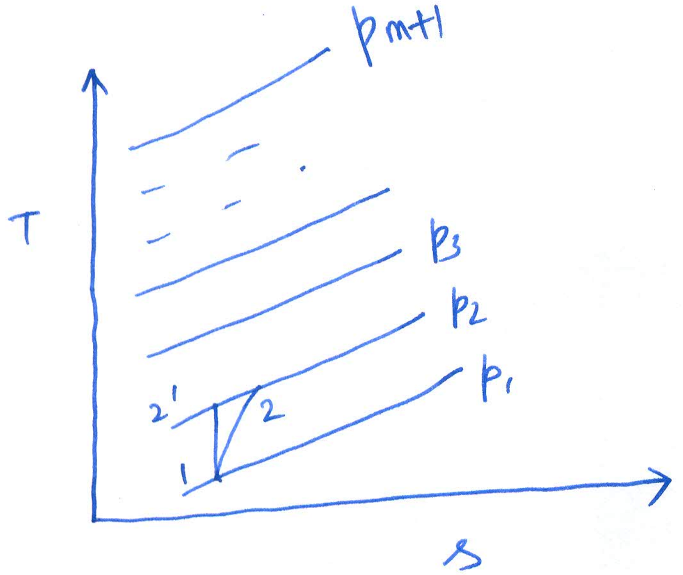
$$w = \eta_t (T_1 - S_5'')$$

$$\eta_s\left[(\;) + (\;) + (\;) + (\;)\right] = \eta_t (T_1 - T_5'')$$

$$\frac{\eta_t}{\eta_s} = \frac{\left[(\;) + (\;) + (\;) + (\;)\right]}{(T_1 - T_5'')} > 1$$

$\boxed{\eta_t > \eta_s}$ Reheating effect

η_t, η_s, p_s, overall ~~pressure~~ pressure ratio



$$\eta_s = \frac{T_2' - T_1}{T_2 - T_1}$$

$$T_2 - T_1 = \frac{T_2' - T_1}{\eta_s}$$

$$= \frac{T_1}{\eta_s}\left(\frac{T_2'}{T_1} - 1\right)$$

$$= \frac{T_1}{\eta_s}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\boxed{\Delta T_1 = \alpha T_1}$$

$$\Delta T_2 = T_3 - T_2 = \frac{T_3' - T_2}{\eta_s} = \frac{T_2}{\eta_s}\left[\frac{T_3'}{T_2} - 1\right]$$

$$= \frac{T_2}{\eta_s}\left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\Delta T_2 = \alpha T_2 \qquad T_2 - T_1 = \alpha T_1$$

$$= \alpha(1+\alpha)T_1 \qquad T_2 = (1+\alpha)T_1$$

$$\Delta T_3 = \alpha(1+\alpha)^2 T_1$$

$$\Delta T_m = \alpha(1+\alpha)^{m-1} T_1 .$$

$$\omega = \sum \omega_s = c_p\left[\Delta T_1 + \cdots + \Delta T_m\right]$$

$$= c_p \alpha T_1 \left[1 + (1+\alpha) + \cdots + (1+\alpha)^{m-1}\right]$$

$$= c_p \alpha T_1 \left[1 . \frac{\{(1+\alpha)^m - 1\}}{\cancel{1}+\alpha-\cancel{1}}\right]$$

$$= c_p T_1 \left[(1+\alpha)^m - 1\right] \; *$$

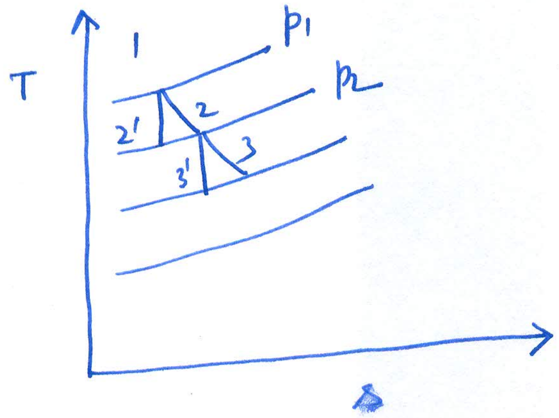
$$\cancel{\eta_t} \quad \eta_c = \frac{h_{m+1}' - h_1}{h_{m+1} - h_1} = \frac{c_p T_1\left[\frac{T_{m+1}'}{T_1} - 1\right]}{c_p T_1 \left[\; * \;\right]} = \frac{\left(\frac{p_{m+1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{[\quad]}$$

$$= \left(\cancel{\frac{p_2}{p_1}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{p_{m+1}}{p_1} = \left(\frac{p_2}{p_1}\right)^m$$

$$\eta_c = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{m(\gamma-1)}{\gamma}} - 1}{1+\cancel{\left(\frac{p_2}{p_1}\right)}\left[(1+\alpha)^m - 1\right]} \qquad \alpha = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_s}$$

<u>Turbine</u>



$$\eta_s = \frac{T_1 - T_2}{T_1 - T_2'}$$

$$\Delta T_1 = \eta_s (T_1 - T_2')$$

$$= \eta_s T_1 \left[1 - \left(\frac{T_2'}{T_1}\right)\right]$$

$$= \eta_s T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$= \beta T_1$$

$$T_1 - T_2 = \beta T_1$$

$$T_2 = (1-\beta) T_1$$

$$\Delta T_2 = \beta T_2$$

$$\eta_t = \frac{1-(1-\beta)^m}{1-\left(\frac{p_2}{p_1}\right)^{\frac{m(\gamma-1)}{\gamma}}}$$