Scaling laws

Prototype Q, H, P Model D, Q, H, P

Pump Q, H

10 lps	70 m	radial
3 m³/s	15 m	mixed flow
		axial

$$\pi_1 = \frac{Q}{ND^3} \qquad \pi_2 = \frac{gH}{N^2D^2} \qquad \pi_3 = \frac{P}{\rho N^3 D^5}$$

Same m/c, unit head. 1m

$$\left.\frac{gH}{N^2D^2}\right|_1 = \frac{gH}{N^2D^2}$$

$$\frac{gH_1}{N_1^2D^2} = \frac{gH}{N^2D^2}$$

$$\frac{N_1^2}{N^2} = \frac{1}{H}$$

$$\boxed{N_1 = N\frac{1}{\sqrt{H}}}$$

Unit speed

$$\left.\frac{Q}{ND^3}\right|_1 = \frac{Q}{ND^3}$$

$$\frac{Q_1}{N_1} = \frac{Q}{N}$$

$$Q_1 = Q\frac{N_1}{N}$$

$$\boxed{Q_1 = Q\frac{1}{\sqrt{H}}}$$

Unit flow rate

Unit speed

$$\left.\frac{P}{\rho N^3 D^5}\right|_1 = \frac{P}{\rho N^3 D^5}$$

$$\frac{P_1}{N_1^3} = \frac{P}{N^3}$$

$$P_1 = P\frac{N_1^3}{N^3}$$

$$\boxed{P_1 = P\frac{1}{H^{3/2}}}$$

Unit power

Ex. Turbine handling water has head 110 m, flow rate $4\ m^3/s$, diameter 1.2 m produces power 3.9 MW at 375 rpm. Find (a) unit ~~power~~ speed, (b) unit flow rate, (c) unit power. (d) ~~sp~~ specific flow rate, (e) specific power

$$N_1 = \frac{N}{\sqrt{H}} = \frac{375}{\sqrt{110}}$$

$$Q_1 = \frac{Q}{\sqrt{H}} = \frac{4}{\sqrt{110}}$$

$$P_1 = \frac{P}{H^{3/2}} = \frac{3.9}{110^{3/2}}$$

$P_a = \rho Q g H$

$= \checkmark$

$P_b =$

Unit head 1m, $D_1 = 1\ m$

$$\left.\frac{gH}{N^2D^2}\right|_1 = \frac{gH}{N^2D^2}$$

$$\frac{N_{11}^2}{N^2} = \frac{D^2}{H}$$

$$\boxed{\frac{N_{11}}{N} = \frac{D}{\sqrt{H}}}$$

$$\left.\frac{Q}{ND^3}\right|_1 = \frac{Q}{ND^3}$$

$$\frac{Q_{11}}{N_1} = \frac{Q}{ND^3}$$

$$Q_{11} = Q \cdot \frac{N_{11}}{N} \cdot \frac{1}{D^3}$$

$$= Q \frac{D}{\sqrt{H}} \cdot \frac{1}{D^3}$$

$$Q_{11} = Q \frac{1}{D^2\sqrt{H}}$$ specific flow rate

$$\left.\frac{P}{\rho N^3 D^5}\right|_1 = \frac{P}{\rho N^3 D^5} \Rightarrow P_{11} = P \frac{N_{11}^3}{N^3} \cdot \frac{D^1}{D^5}$$

$$= P \frac{D^3}{H^{3/2}} \cdot \frac{1}{D^5}$$

$$\boxed{P_{11} = P \frac{1}{D^2 H^{3/2}}}$$ specific power

$$Q_{11} = \frac{4}{1.2^2\sqrt{110}}$$

$$P_{11} = \frac{3.9}{1.2^2\,110^{3/2}}$$

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$$\pi_1 = \frac{Q}{ND^3}\ ; \quad \pi_2 = \frac{gH}{N^2D^2}\ ; \quad \pi_3 = \frac{P}{\rho N^3 D^5}$$

$$N_{11} = \frac{ND}{\sqrt{H}} \qquad ND \sim U$$

$$\sqrt{H} \sim V$$

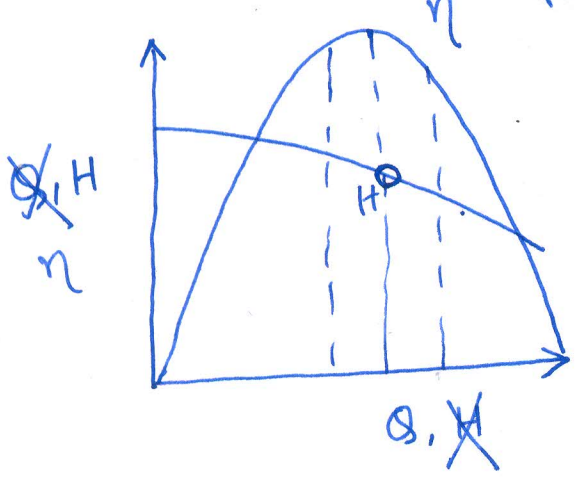
ϕ = velocity ratio

$$\pi_2 = \frac{gH}{N^2D^2} \sim \frac{1}{\phi^2}$$

$$\pi_1 = \frac{Q}{D^2 ND} \sim \frac{V}{U} \sim \frac{1}{\phi}$$

$\pi_3 =$

BEP = Best efficiency Point



~~Performance characteristic~~

Head - Flow characteristics

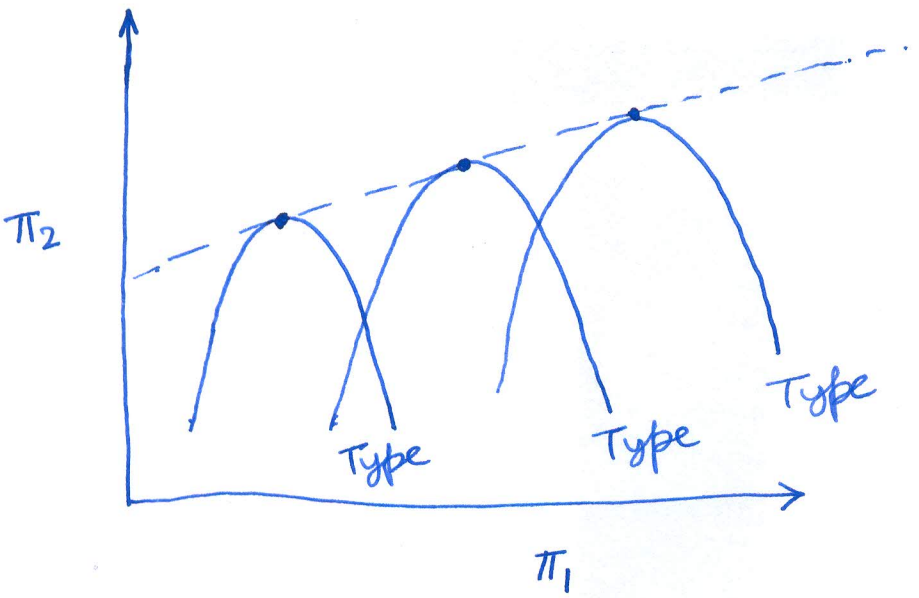
Pump

Backward - curved vane

$$\text{Power} = \frac{\rho Q g H}{\eta}$$

Q, H

P, H,



Type radial
mixed
axial

D

$$\pi_1 = \frac{Q}{ND^3}\ ;\ \pi_2 = \frac{gH}{N^2D^2}\ ;\ \pi_3 = \frac{P}{\rho N^3 D^5}$$

$$\pi_4 = \left(\frac{\pi_1^2}{\pi_2^3}\right)^{1/4} = \left(\frac{Q^2}{N^2D^6} \times \frac{N^6D^6}{(gH)^3}\right)^{1/4} = \underbrace{\left(N^4 \frac{Q^2}{(gH)^3}\right)^{1/4}}_{N_{SP}} = \boxed{N\frac{\sqrt{Q}}{(gH)^{3/4}}}$$

$$\pi_5 = \left(\frac{\pi_3^2}{\pi_2^5}\right)^{1/4} = \left(\frac{P^2}{\rho^2 N^6 D^{10}} \times \frac{N^{10}D^{10}}{(gH)^5}\right)^{1/4} = \left(N^4 \frac{P^2}{\rho^2 gH^5}\right)^{1/4}$$

K_{ST} N_{ST}

$$= \boxed{N\frac{\sqrt{P}}{\sqrt{\rho}\,(gH)^{5/4}}}$$

Shape number
Type number

Turbine 110m

$H = 110m$

$Q = 4 m^3/s$ $\quad N = 375$ rpm.

$P = 3.9$ MW

$P_a = 1000 \times 4 \times 9.81 \times 110$ W

$\simeq 4.3$ MW

$$N_{ST} = N \times \frac{\sqrt{P}}{\sqrt{\rho}\,(gH)^{5/4}}$$

$$= \left(\frac{375}{60}\right) \times \frac{\sqrt{3.9 \times 10^6}}{\sqrt{1000} \times (9.81 \times 110)^{5/4}}$$

~~rpm~~ rev/sec
~~rad/sec~~

$=$

Ex 12 A turbine of speed 375 rpm is designed for a project site where the available head and flow rate are 35m of water and $15 m^3/s$. The model of this turbine runs at 250 rpm under a head of 2m. Calculate the specific speed, scale ratio, flow rate and power of the model. Find the specific speed (type number) of prototype and model.

.303 2.79 .460 9.026 kW

N_{ST} N_{ST}