PART A

Q2. Uncertainity of bending moment, $M = \pm 15\%$ ($n_d = \frac{\text{Loss of function Parameter}}{\text{Allowable Parameter}}$)

Bending moment causing failure = 200 ± 40 Nm

$\therefore$ Uncertainity in bending moment causing failure = $\pm 20\%$

$\therefore$ Design factor when there is uncertainity in bending moment causing failure

$$n_d = \frac{0.8\, M_{failure}}{M_{allowable}}$$

0.8 — 1.2

↑ Min

$\Rightarrow M_{allowable} = 0.8\, M_{failure}/n_d$

Design factor when there is uncertainity in bending moment

$$n_d = \frac{M_{failure}}{1.15\, M_{allowable}}$$

0.85 — 1.15

↑ Max

$\Rightarrow M_{failure} = n_d \times 1.15\, M_{allowable}$

$\therefore$ Design factor accounting for both uncertainities

$$n_d = \frac{\frac{1}{0.8}}{\frac{1}{1.15}}$$

$$= \frac{1.15}{0.8}$$

$$\boxed{n_d = 1.4375}$$

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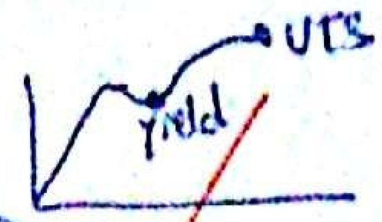
$\therefore \quad n_d = \frac{M_{\text{loss of function}}}{M_{allowable}}$

$\Rightarrow M_{allowable} = \frac{200}{1.4375}$ Nm

$\Rightarrow \boxed{M_{allowable} = 139.13 \text{ Nm}}$

a) True (

)

b) True (stress = $\frac{\text{Applied Force}}{\text{Area}}$)

c) False

d) False

e) False ($\frac{E}{\rho^{1/2}}$) must be minimized

f) True

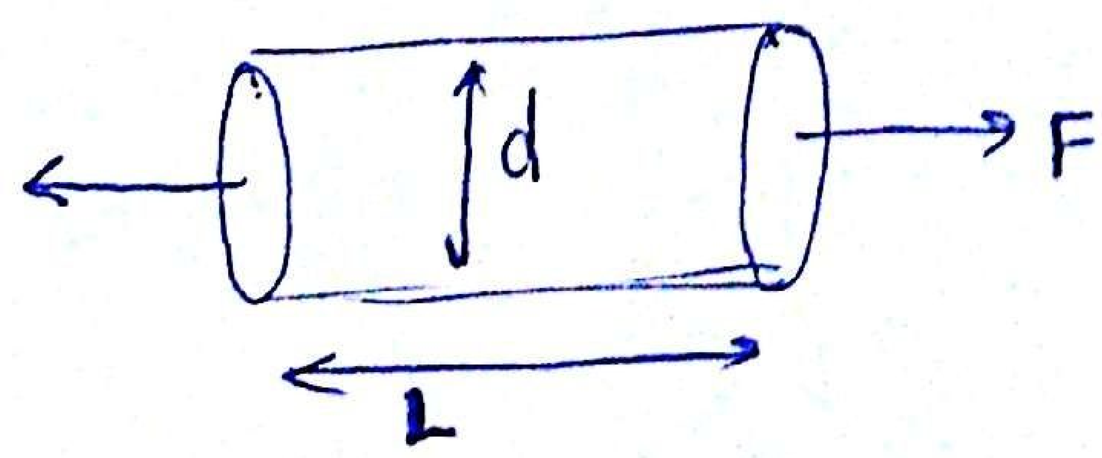
g) False

h) False $\tau = \frac{Tr}{J} \Rightarrow \tau \propto r$

i) False $\delta = \frac{CPL^3}{EI}$ $\therefore$ No hardness term

j) False 6 constants are needed

Q4.



$\therefore m = \rho A L$ — (i)

$\frac{F}{A} < \sigma_y$

$\Rightarrow A > \frac{F}{\sigma_y}$ — (ii)

from (i) and (ii), we have

$m = \rho A L$

$\therefore \frac{m}{\rho L} > \frac{F}{\sigma_y}$

$\Rightarrow m > FL\left(\frac{\rho}{\sigma_y}\right)$

At the limiting condition,

$$\boxed{m = FL\left(\frac{\rho}{\sigma_y}\right)}$$

$$\therefore \frac{m}{L} = \left(\frac{F\rho}{\sigma_y}\right)$$

	ρ (kg/m3)	σ_y (MPa)	$(m/L)_{min}$ (kg/m)
1020 CD steel	7860	390	10.076
4140 Q&T at 425°C	7860	$\boxed{1140}$	3.447
Ti-6Al-4V	$\boxed{4460}$	850	$\boxed{2.623}$
AA 2017-0	2790	70	19.928
Tungsten	19300	750	12.866

Ti-6Al-4V results in the lowest mass per unit length needed to support the stated load.

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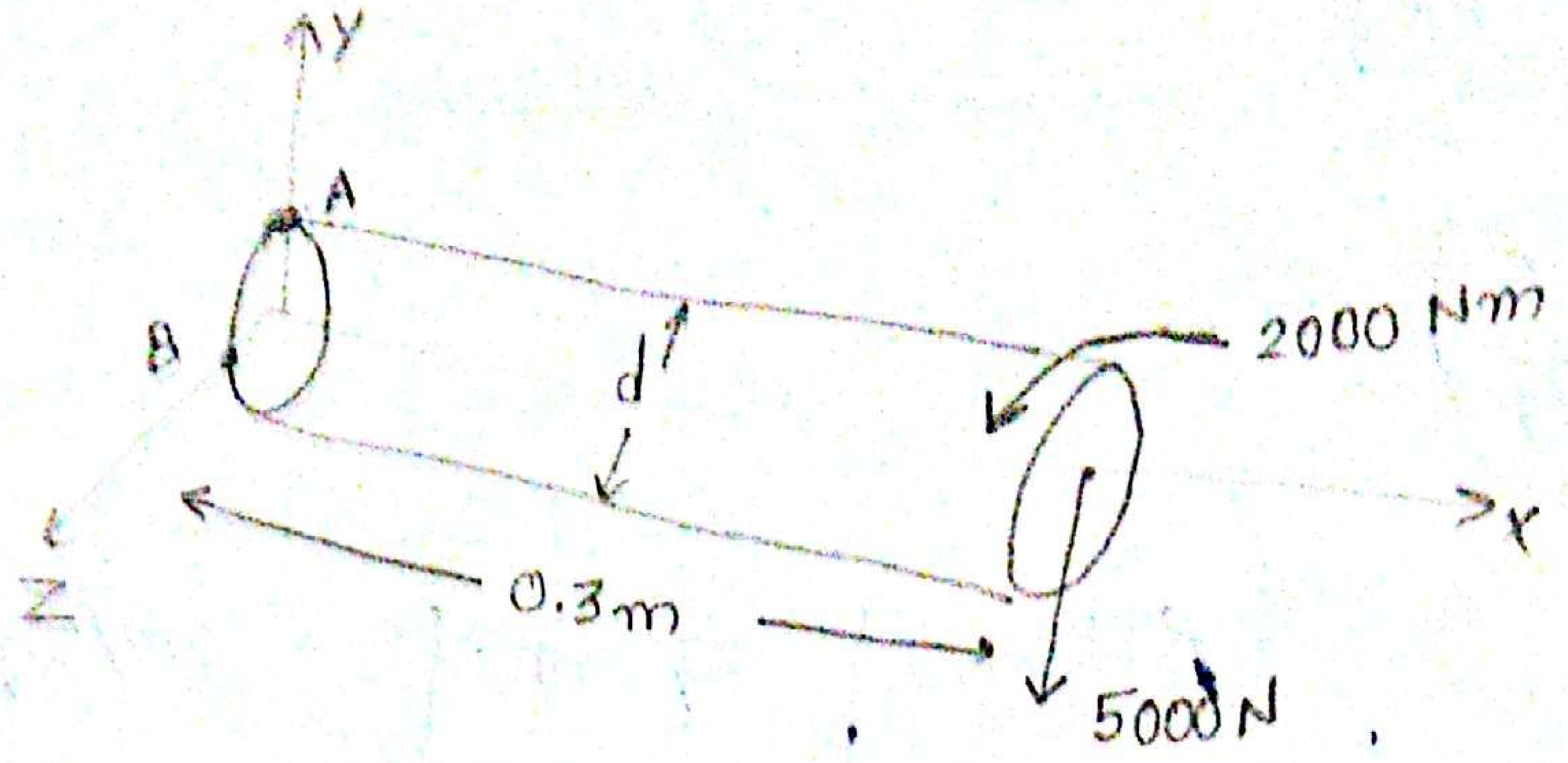
PART B

Q.B



FOS = 2.5

S_y = 320 MPa

We examine the situation at two points A and B which are expected to be in maximum stress.

For the point A,

$$\tau_{Torsion} = \frac{16T}{\pi d^3} = \frac{16 \times 2000}{\pi d^3} = \frac{10185.9}{d^3}$$ (xz plane, +z direction)

$$\sigma_{Bending} = \frac{32M}{\pi d^3} = \frac{32 \times 5000 \times 0.3}{\pi d^3} = \frac{15278.8}{d^3}$$ (x axis)

$\tau_{shear} = 0$ (as the point A is at periphery in the direction of applied force)

For the point B,

$$\tau_{Torsion} = \frac{10185.9}{d^3}$$ (xy plane, -y direction)

$\sigma_{Bending} = 0$ (as the point B is at the Neutral Axis)

$$\tau_{shear} = \frac{4V}{3A} = \frac{4 \times 5000 \times 4}{3 \pi d^2} = \frac{8488.26}{d^2}$$ (xy plane, -y direction)

Using Mises criterion for A, we have

$$\sigma_A' = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right)^{1/2}$$

$$= \left(\left(\frac{15278.8}{d^3}\right)^2 + 0 + 0 + 3 \times \left(\frac{10185.9}{d^3}\right)^2\right)^{1/2}$$

$$= \frac{23338.8}{d^3}$$

$$\therefore \frac{23338.8}{d^3} < \frac{320 \times 10^6}{2.5}$$

$$\Rightarrow d^3 > 1.823 \times 10^{-4}\ m^3$$

$$\Rightarrow d > 0.0567\ m$$

$$\Rightarrow d > 56.705\ mm$$

$$\Rightarrow \boxed{d \approx 57\ mm}$$

Using Mises criterion for B, we have

$$\sigma_B' = \left(3 \times \left(\frac{10185.9}{d^3} + \frac{8488.26}{d^2}\right)^2\right)^{1/2}$$

$$= \sqrt{3}\left(\frac{10185.9}{d^3} + \frac{8488.26}{d^2}\right)$$

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Putting $d = 57\ mm$,

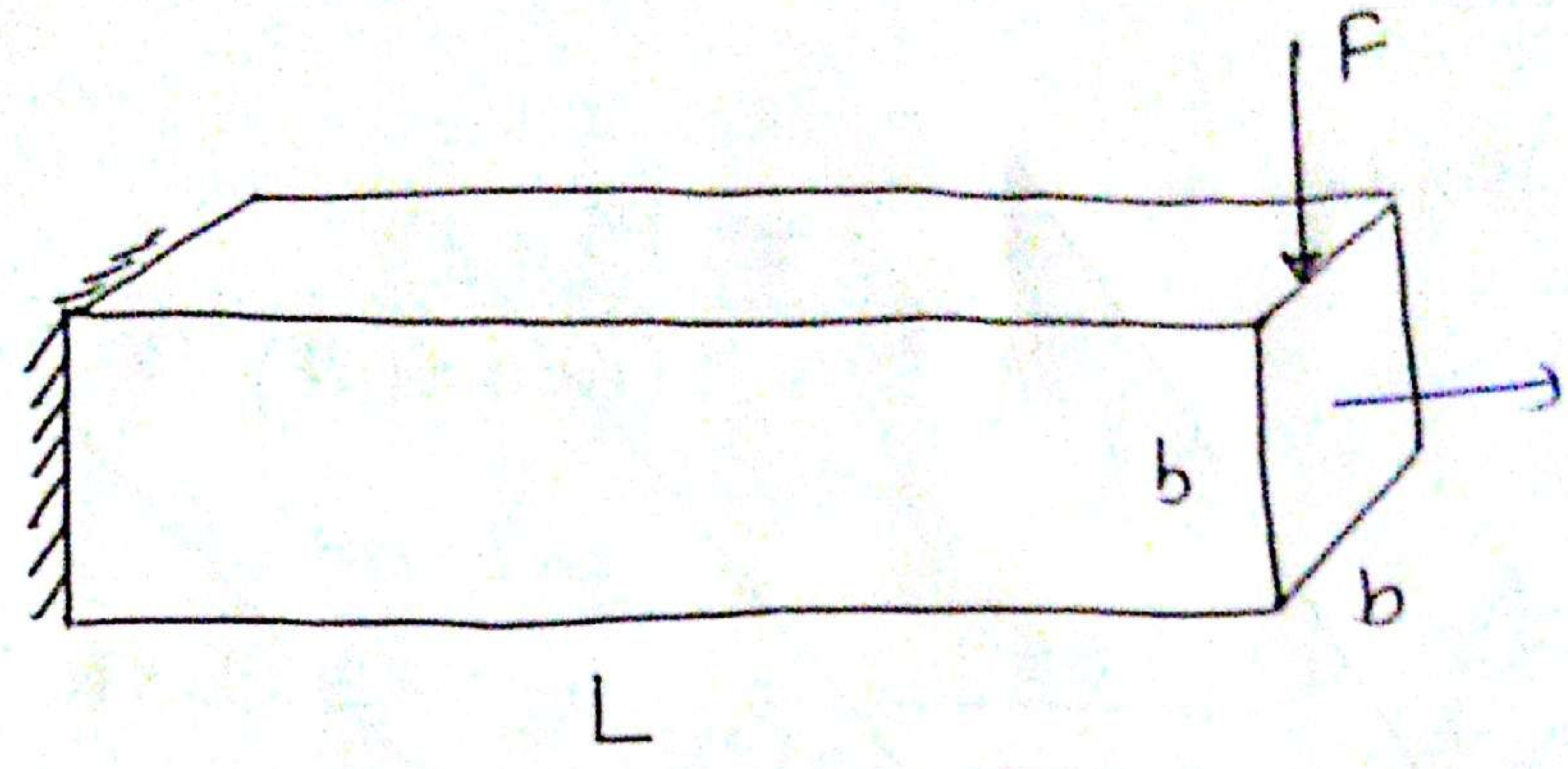
we get $\sigma_B' \approx 99.8$ MPa

which is less than $\frac{320}{2.5} = 120$ MPa

$\therefore$ ~~σ_A is the~~

A is the critical point and hence the diameter is greater than or equal to 57 mm (approx)

Ques 6



$M = F \cdot L$ $\qquad I = \frac{1}{12} b^4$

$$\sigma_x = \frac{My}{I}$$

$$\sigma_x = \frac{F \cdot L \cdot b/2}{\frac{1}{12} b^4}$$

$$\sigma_x = \frac{6FL}{b^3}$$

$$\sigma_x \leq \sigma_y$$

$$\Rightarrow m = L \cdot b^2 \rho$$

$$b^3 = \frac{6FL}{\sigma_y}$$

$$b = \left(\frac{6FL}{\sigma_y} \right)^{1/3}$$

$$\Rightarrow m = L\rho \cdot \left(\frac{6FL}{\sigma_y} \right)^{2/3}$$

$$m = L \cdot (6FL)^{2/3} \cdot \frac{\rho}{\sigma_y^{2/3}}$$

$m \propto \frac{\rho}{\sigma_y^{2/3}}$ (minimize the mass)

① Tungsten carbide $= \frac{15200}{(370 \times 10^6)^{2/3}} = 0.0295$

$= \frac{15200}{(3350 \times 10^6)^{2/3}} = 6.79 \times 10^{-3}$

② AISI 1095 $= \frac{7860}{(765 \times 10^6)^{2/3}} = 9.397 \times 10^{-3}$

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③ AA7075T6 $= \frac{2700}{(540 \times 10^6)^{2/3}} = 4.0716 \times 10^{-3}$

④ Polycarbonate $= \frac{1270}{(120 \times 10^6)^{2/3}} = 5.2201 \times 10^{-3}$

So that AA7075T6 has minimum mass

so our requirement completed by AA7075T6 cause of failure =)

yielding

$$\sigma_x = \frac{My}{I}$$

shearing

$$\tau = \frac{3F}{2A}$$

Q7

This problem can be modelled as:

from energy balance equations, we get the force exerted as

$$F = W\left(1+\sqrt{1+\frac{2kh}{W}}\right) \quad \text{where } W = mg$$

$$\boxed{\therefore F_{max} = mg\left(1+\sqrt{1+\frac{2kh}{mg}}\right)}$$

for the bar, we have

$$\Delta l = \frac{FL}{AE}$$

We know, $\frac{F}{\Delta l} = K$

$$\boxed{\therefore K = \frac{AE}{L}}$$

Here, for the bar, we have $A = \frac{\pi d^2}{4}$ $\therefore K = \frac{\pi d^2 E}{4L}$

$E = 205$ GPa

$L = L$

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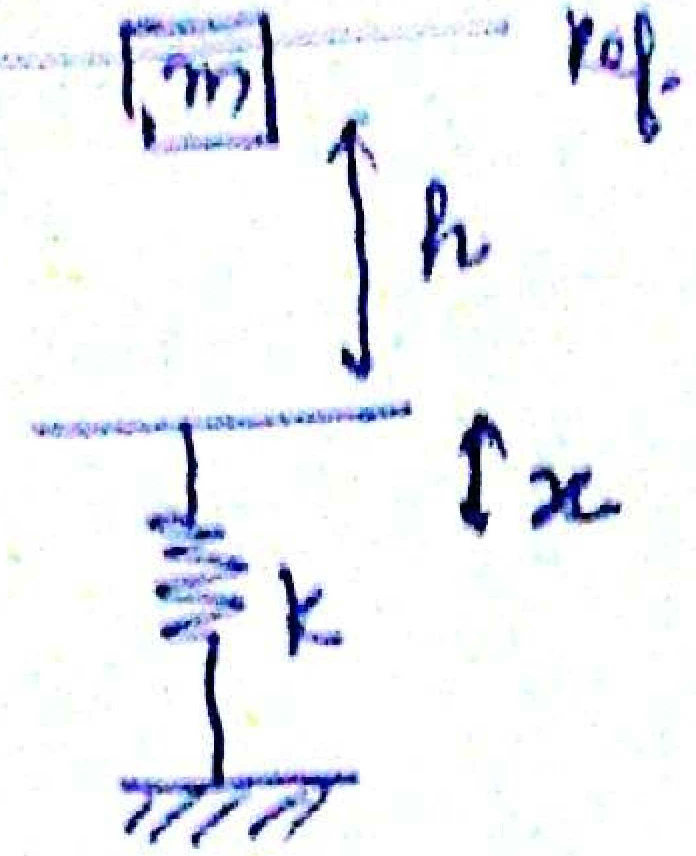
$$\therefore F_{max} = mg\left(1+\sqrt{1+\frac{2\pi d^2 E h}{4L\, mg}}\right)$$

$$F_{max} = mg\left(1+\sqrt{1+\frac{\pi d^2 E h}{2Lmg}}\right)$$ where $E = \cancel{210}$ 205 GPa

$g \rightarrow$ gravitational acceleration

Proof



$$\therefore \Delta KE = -\Delta PE$$

$$\Rightarrow 0 - 0 = -\left(-mg(x+h) + \tfrac{1}{2}kx^2\right)$$

$$\Rightarrow mgx + mgh = \tfrac{1}{2}kx^2$$

$$\Rightarrow kx^2 - 2mgx - 2mgh = 0$$

$$\therefore x = \frac{2mg \pm \sqrt{(2mg)^2 + 4k.2mgh}}{2k}$$

$$= \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1 + \frac{2kh}{mg}}$$

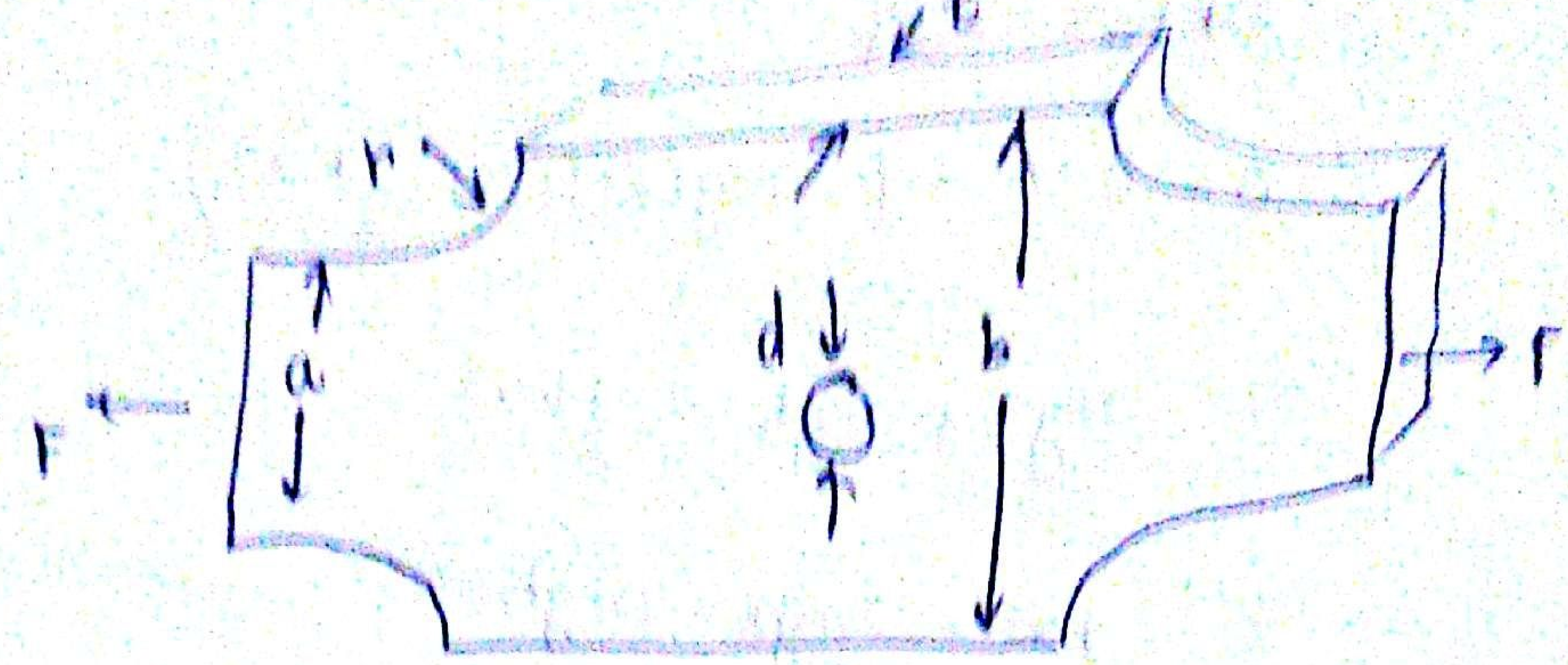
$$= \frac{mg}{k}\left(1 + \sqrt{1 + \frac{2kh}{mg}}\right)$$

(+ sign for maximum force

$$\therefore F_{max} = kx$$

$$= mg\left(1 + \sqrt{1 + \frac{2kh}{mg}}\right)$$

So



For the section at the hole, we have

$$\sigma = \frac{F}{t\,(b-d)}$$

$$= \frac{80{,}000}{t\,(0.09-0.02)} \text{ Pa}$$

$$= \frac{1.142}{t} \text{ MPa}$$

for the stress concentration factor, we have

$$\frac{d}{w} = \frac{20}{90} = 0.22$$

∴ stress concⁿ. factor $k_f = 2.45$

$$\therefore \sigma_{max} = \frac{2.8}{t} \text{ MPa}$$

For the section at the fillet, we have

$$\sigma = \frac{F}{ta}$$

$$= \frac{80,000}{t \times 0.06} \text{ Pa}$$

$$= \frac{1.333 \text{MPa}}{t}$$

for the stress concentration, we have

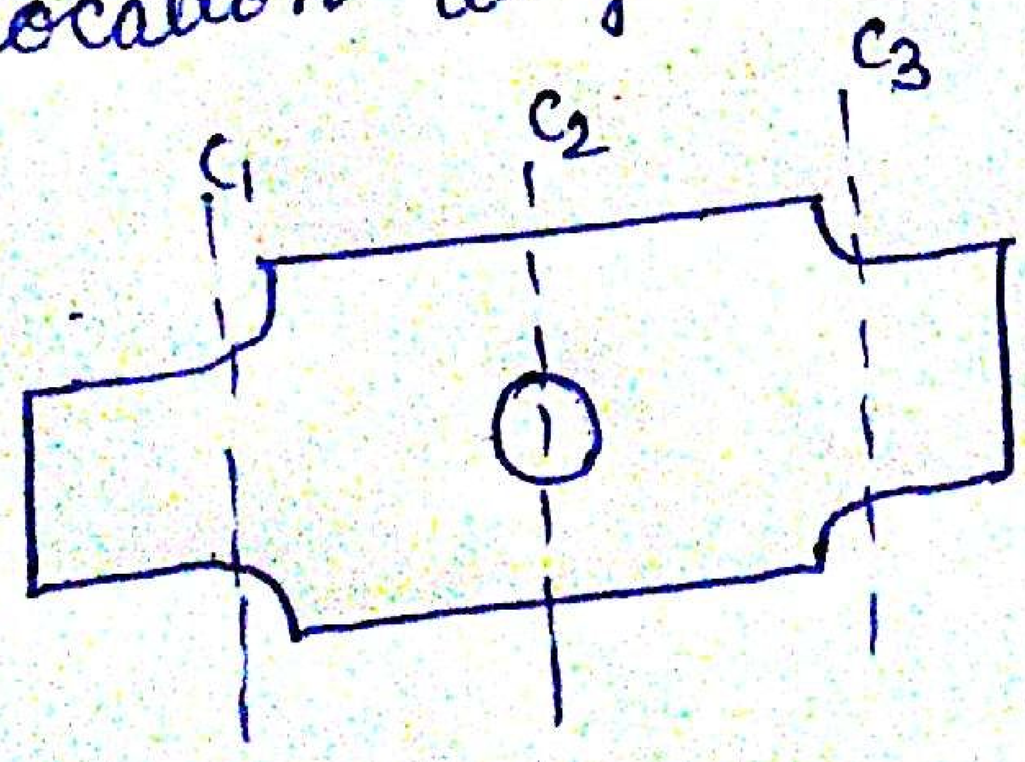
$$\frac{r}{d} = \frac{6}{60} = 0.1 \quad ; \quad \frac{D}{d} = \frac{90}{60} = 1.5$$

$\therefore$ stress concentration factor. $K_f = 2.1$

$$\therefore \sigma_{max} = \frac{2.8}{t} \text{ MPa}$$

So, the cross-sections at the hole and at the fillets are equally likely to be critical.

Critical location diagram



To determine minimum thickness,

$$\sigma_{max} \leq \frac{\sigma_{ys}}{\eta}$$

$$\Rightarrow \frac{2.8}{t} \leq \frac{360}{3}$$

$$\Rightarrow t \geq 0.0233 \text{ m}$$

$$\Rightarrow t \geq 23.33 \text{ mm}$$

$$\Rightarrow \boxed{t \approx 24 \text{ mm}}$$

Figure A-15-1

Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

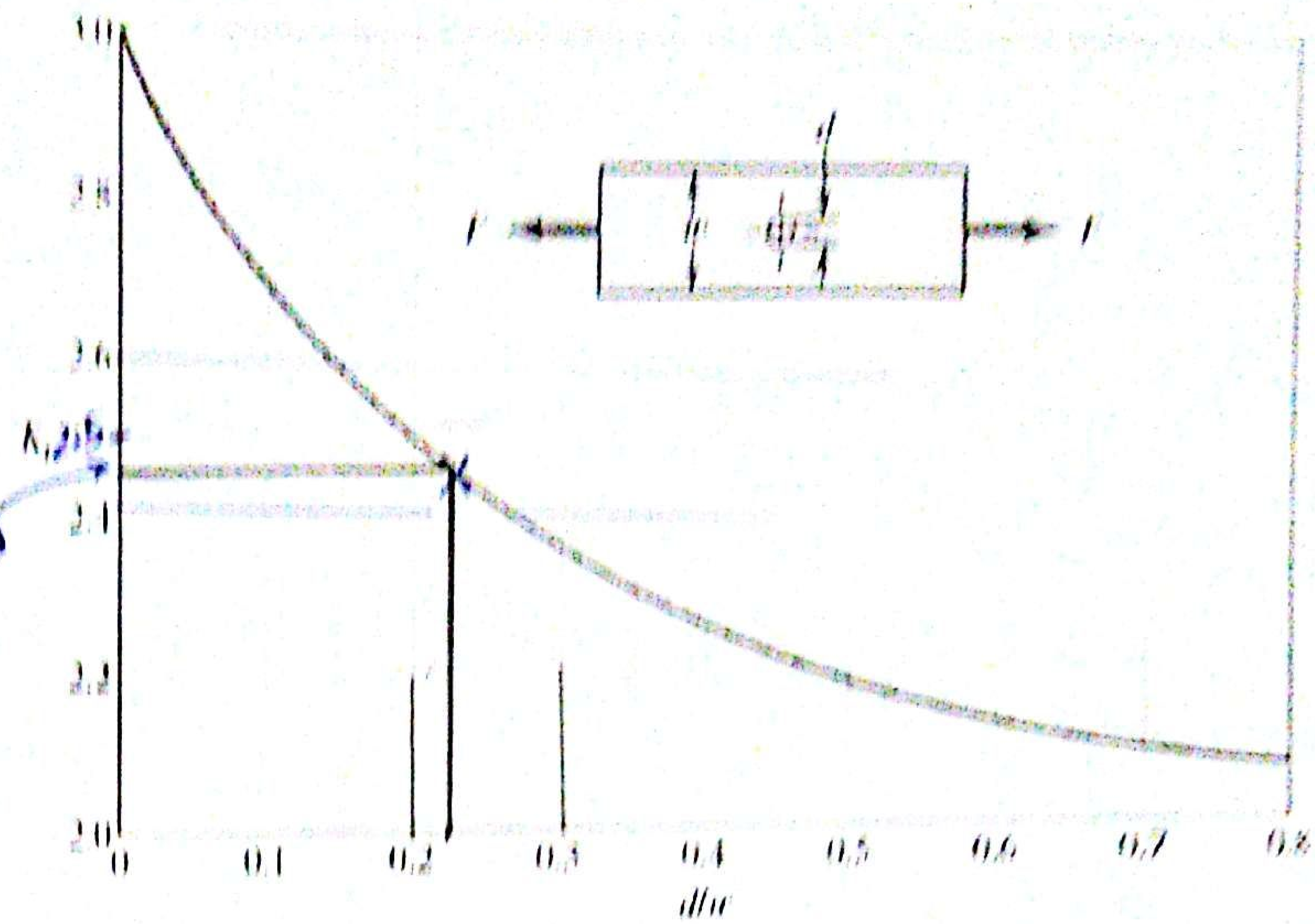
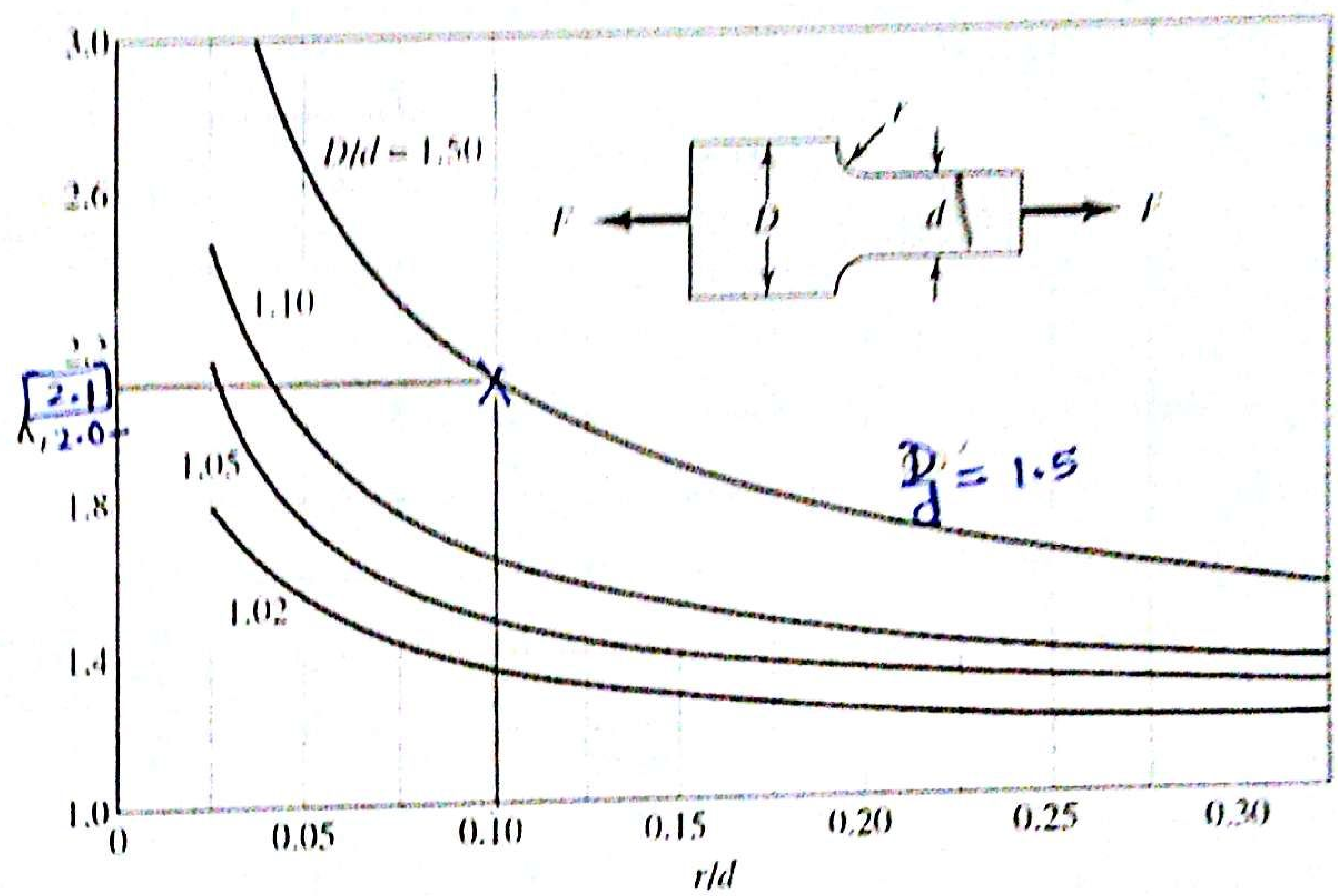


Figure A-15-5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



$\frac{r}{d} = 0.1$