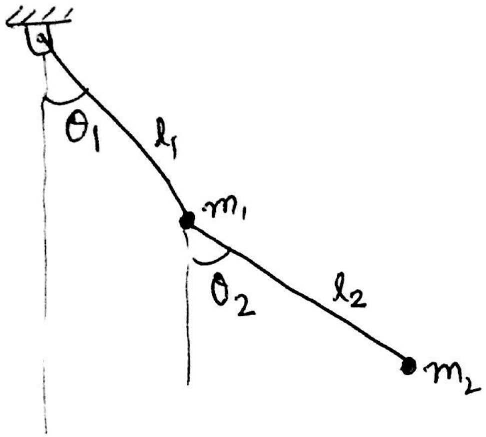
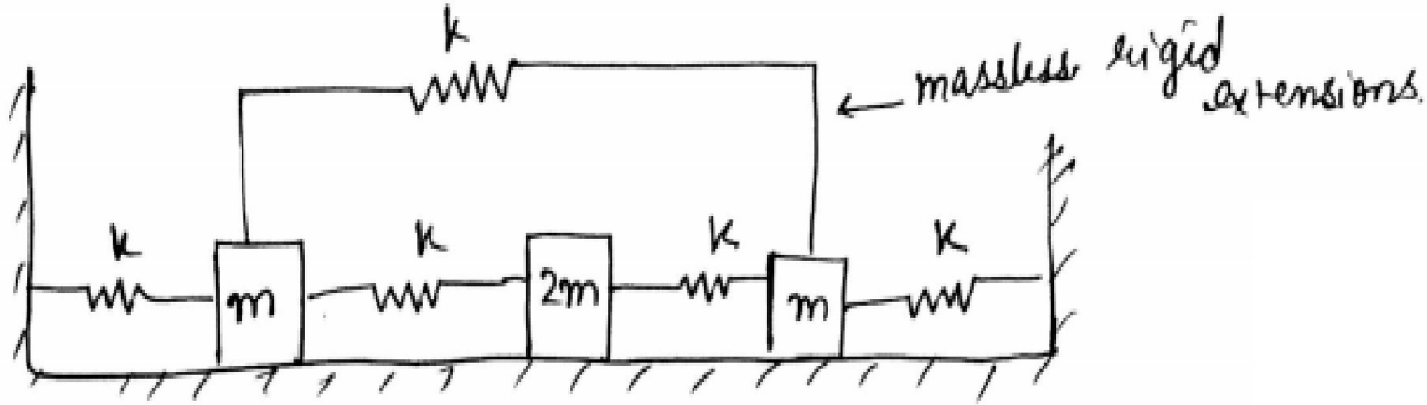
TUTORIAL

Q1

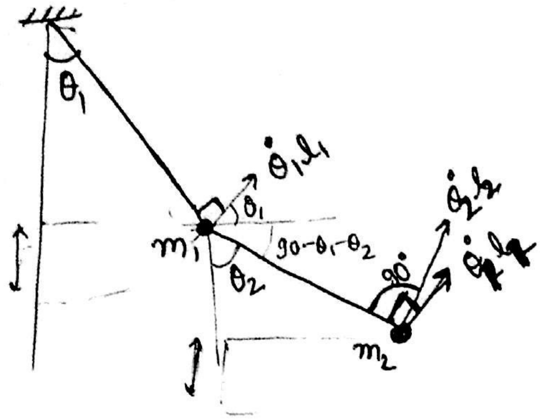


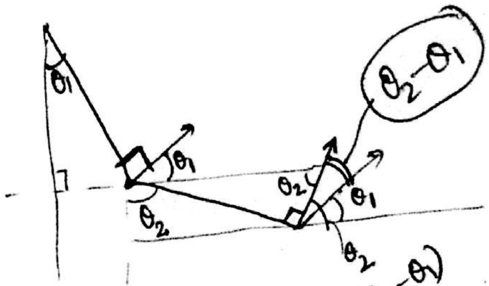
Obtain the non linear DEOM and linearize them stating conditions. Use lagrange equations.

Q2 For the problem of previous Tut, obtain the natural frequency and modal vectors by the MI method. Start with ω_1 and $\{A_1\}$. Iterate till the exact.



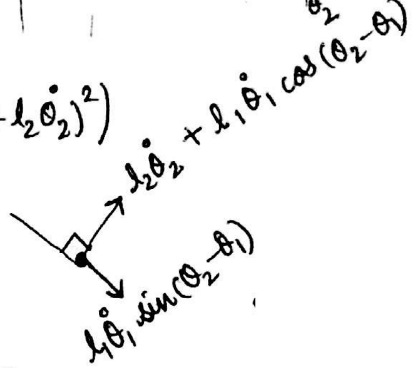
Q1.





We have $T = \frac{1}{2} m_1 (l_1\dot\theta_1)^2$

$$+ \frac{1}{2} m_2 \left(\left(l_1\dot\theta_1 \sin(\theta_2-\theta_1)\right)^2 + \left(l_1\dot\theta_1\cos(\theta_2-\theta_1) + l_2\dot\theta_2\right)^2 \right)$$



$$\Rightarrow T = \frac{1}{2} m_1 l_1^2 \dot\theta_1^2$$

$$+ \frac{1}{2} m_2 \left(l_1^2\dot\theta_1^2 \sin^2(\theta_2-\theta_1) + l_1^2\dot\theta_1^2\cos^2(\theta_2-\theta_1) + l_2^2\dot\theta_2^2 + 2 l_1 l_2 \dot\theta_1\dot\theta_2 \cos(\theta_2-\theta_1) \right)$$

$$\Rightarrow T = \frac{1}{2} m_1 l_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 l_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 l_2^2\dot\theta_2^2 + m_2 l_1 l_2 \dot\theta_1\dot\theta_2\cos(\theta_2-\theta_1)$$

$$\Rightarrow T = \frac{(m_1+m_2)}{2} l_1^2\dot\theta_1^2 + \frac{1}{2} m_2 l_2^2 \dot\theta_2^2 + m_2 l_1 l_2 \dot\theta_1\dot\theta_2 \cos(\theta_2-\theta_1)$$

$$U = m_1 g l_1 (1-\cos\theta_1) + m_2 g\left[l_2(1-\cos\theta_2) + l_1(1-\cos\theta_1)\right]$$

$$= m_1 g l_1 (1-\cos\theta_1) + m_2 g l_2 (1-\cos\theta_2) + m_2 g l_1 (1-\cos\theta_1)$$

$$U = (m_1+m_2) g l_1 (1-\cos\theta_1) + m_2 g l_2 (1-\cos\theta_2)$$

From Lagrange Eq^n,

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial\dot\theta_1}\right) - \frac{\partial T}{\partial\theta_1} + \frac{\partial U}{\partial\theta_1} = 0$$

$$\frac{\partial T}{\partial\dot\theta_1} = (m_1+m_2) l_1^2\dot\theta_1 + m_2 l_1 l_2 \dot\theta_2\cos(\theta_2-\theta_1)$$

$$\frac{\partial T}{\partial\theta_1} = + m_2 l_1 l_2 \dot\theta_1\dot\theta_2 \sin(\theta_2-\theta_1)$$

$$\frac{\partial U}{\partial\theta_1} = (m_1+m_2) g l_1 \sin\theta_1$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial\dot\theta_2}\right) - \frac{\partial T}{\partial\theta_2} + \frac{\partial U}{\partial\theta_2} = 0$$

$$\frac{\partial T}{\partial\dot\theta_2} = m_2 l_2^2\dot\theta_2 + m_2 l_1 l_2 \dot\theta_1\cos(\theta_2-\theta_1)$$

$$\frac{\partial T}{\partial\theta_2} = - m_2 l_1 l_2 \dot\theta_1\dot\theta_2\sin(\theta_2-\theta_1)$$

$$\frac{\partial U}{\partial\theta_2} = m_2 g l_2 \sin\theta_2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot\theta_1}\right) = (m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1)$$
$$+ m_2 l_1 l_2 \dot\theta_2 \left(-\sin(\theta_2-\theta_1)\left(\dot\theta_2-\dot\theta_1\right)\right)$$
$$= (m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1) - m_2 l_1 l_2 \dot\theta_2^2 \sin(\theta_2-\theta_1) + m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot\theta_2}\right) = m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \cos(\theta_2-\theta_1) + m_2 l_1 l_2 \dot\theta_1 \left(-\sin(\theta_2-\theta_1)\left(\dot\theta_2-\dot\theta_1\right)\right)$$
$$= m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \cos(\theta_2-\theta_1) - m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)$$
$$+ m_2 l_1 l_2 \dot\theta_1^2 \sin(\theta_2-\theta_1)$$

∴ DEOM 1

$$(m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1) - m_2 l_1 l_2 \dot\theta_2^2 \sin(\theta_2-\theta_1) + \cancel{m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)}$$
$$- \cancel{m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)} + (m_1+m_2) g l_1 \sin\theta_1 = 0$$

DEOM 2

$$m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \cos(\theta_2-\theta_1) + m_2 l_1 l_2 \dot\theta_1^2 \sin(\theta_2-\theta_1) + m_2 g l_2 \sin\theta_2 = 0$$

The angles θ_2 and θ_1 are small $(\sin\theta_1 \approx \theta_1 ; \sin\theta_2 \approx \theta_2)$

∴ $\dot\theta_1^2 \sin(\theta_2-\theta_1)$ and $\dot\theta_2^2 \sin(\theta_2-\theta_1)$ can be neglected

∴ DEOM 1

$$(m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 + (m_1+m_2) g l_1 \theta_1 = 0 \quad \text{— (I)}$$
$$m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 + m_2 g l_2 \theta_2 = 0 \quad \text{— (II)}$$