Tu/HW (Lagrange's Eqns)

71. A double pendulum is connected by four springs of equal stiffness as shown in Fig. 2-73 below. For small angles of oscillation, find its frequencies by the use of Lagrange's equation.

Ans. $\omega_2 = \sqrt{2k/m + 3.12g/L}$, $\omega_1 = \sqrt{2k/m + 0.58g/L}$ rad/sec

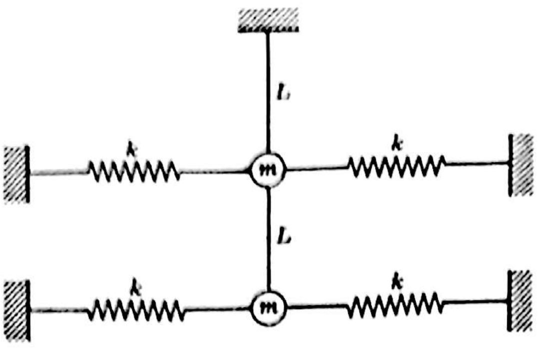


Fig. 2-73

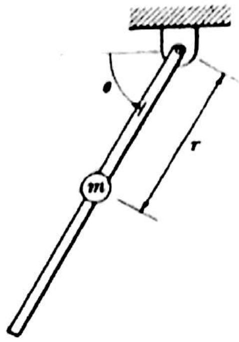


Fig. 2-74

72. A small mass m is free to slide on a homogeneous uniform rod of mass M and length L which is pivoted at one end as shown in Fig. 2-74 above. Derive the equations of motion by the use of Lagrange's equation.

Ans. $(ML^2 + mr^2)\ddot{\theta} + 2mr\dot{r}\dot{\theta} - (mr + ML)g\cos\theta = 0$

$m\ddot{r} - m\dot{\theta}^2 r + mg(1 - \sin\theta) = 0$

73. A circular homogeneous cylinder of mass M and radius R rolls without slipping inside a circular surface of radius $3R$. A small mass m, connected by two equal springs of modulus k, is initially at the center of the cylinder at the equilibrium position as shown in Fig. 2-75 below. Derive expressions for the equations of motion of the system by the use of Lagrange's equation.

Ans. $4(MR^2 + J_o + mR^2)\ddot{\theta} + 2(M + m)gR\theta + 2mR\ddot{r} - 2mgr = 0$

$m\ddot{r} + 2kr + 2mR\ddot{\theta} - 2mg\theta = 0$

where J_o is the moment of inertia of the cylinder.

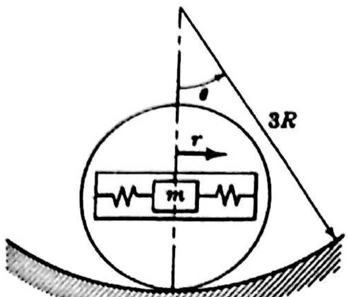


Fig. 2-75

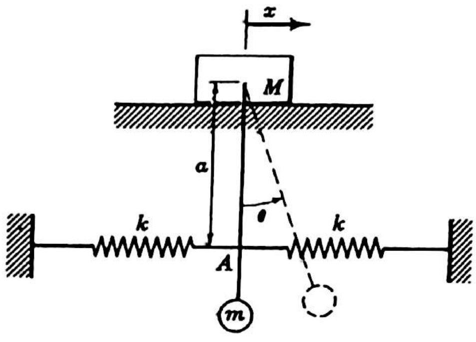


Fig. 2-76

74. A particle of mass m is moving on a horizontal plane under the action of an attractive force which is a function of the displacement, i.e. $F(t) = f(1/r^2)$. Determine the equations of motion of the particle by the use of Lagrange's equation.

k/r^2

Ans. $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$m\ddot{r} + k/r^2 - mr\dot{\theta}^2 = 0$

75. The block of mass M moves along a smooth horizontal plane, and carries a simple pendulum of length L and mass m as shown in Fig. 2-76 above. Two equal springs of modulus k are connected to the pendulum at point A. Determine the equations of motion describing small oscillation of the system about the equilibrium position by the use of the Lagrange's equation.

Ans. $(M + m)\ddot{x} + 2kx + mL\ddot{\theta} + 2ak\theta = 0$

$mL^2\ddot{\theta} + (mgL + 2ka^2)\theta + mL\ddot{x} + 2akx = 0$

(PTO)

Tu/HW (Lagrange's Equn.s)

19. Use Lagrange's equation to derive the equations of motion for the coupled pendulum as shown in Fig. 2-23.

21. A double pendulum of lengths L_1 and L_2, masses m_1 and m_2 is shown in Fig. 2-25. Use Lagrange's equation to derive the equations of motion.

26. A circular cylinder of radius r and mass m rolls without slipping inside a semi-circular groove of radius R. Block M is supported by a spring of constant k and constrained to move without friction in the vertical guide as shown in Fig. 2-30. By the use of Lagrange's equation, find the equations of motion of the system.

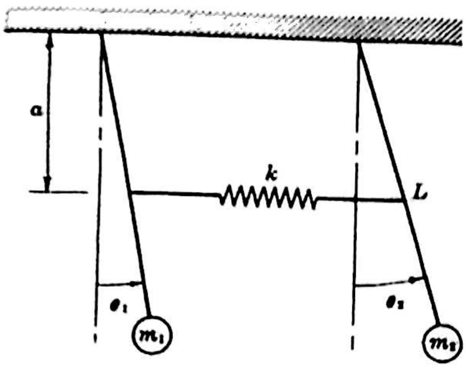


Fig. 2-23

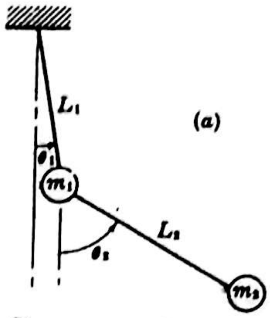


Fig. 2.25

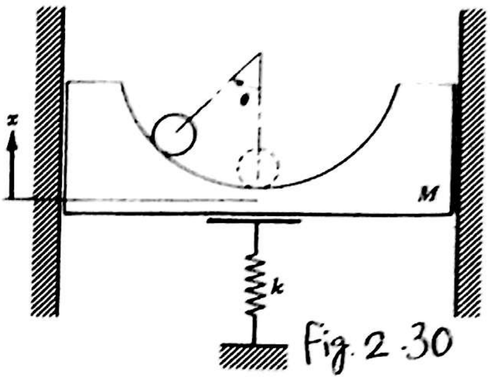
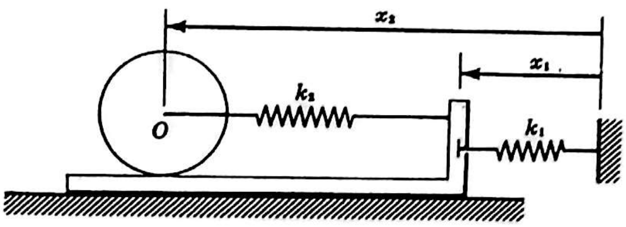


Fig. 2.30

25. A solid homogeneous cylinder of mass M and radius r rolls without slipping on a cart of mass m as shown in Fig. 2-29. The cart, connected by springs of constants k_1 and k_2, is free to slide on a horizontal surface. By the use of Lagrange's equation, find the equations of motion of the system.



27. Fig. 2-32 shows a two-degree-of-freedom spring-mass system with damping. Determine the equations of motion of the system by the use of Lagrange's equation.

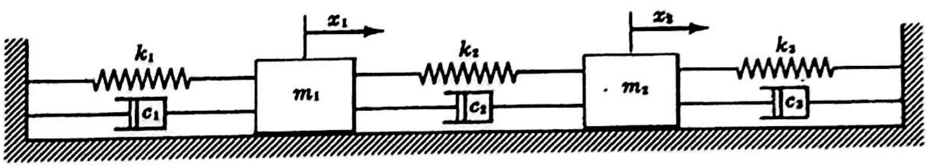
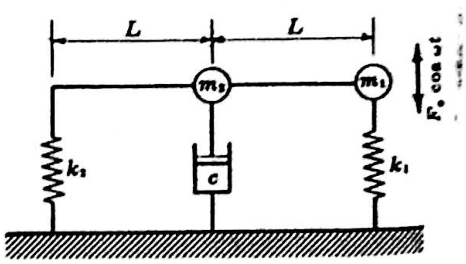


Fig. 2-32

29. Two masses m_1 and m_2 are attached to a rigid weightless bar which is supported by springs k_1 and k_2 and dashpot c as shown in Fig. 2-34. If the motion of the bar is restricted to the plane of the paper, determine the equations of motion of the system by the use of Lagrange's equation.



(PTO)