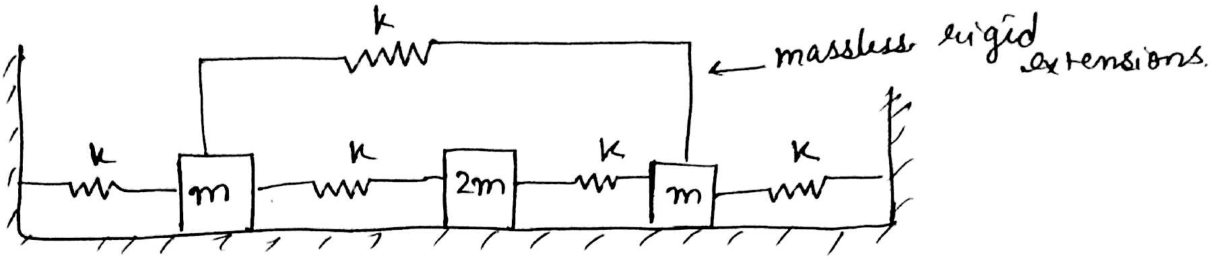
Tutorial on flexible infl. coefficient errors in ω and Rayleigh method

(1)



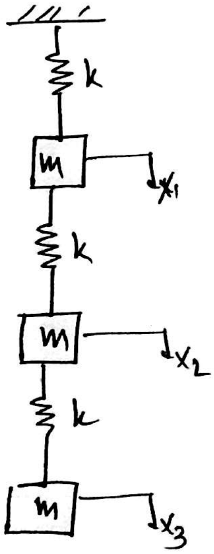
(a). Obtain flexibility matrix [a] from definition of a_{ij}.

(b) Obtain [k] using definition of k_{ij}.

(c) Obtain $\omega_R (\approx \omega_1)$ by the Rayleigh method.

For part (c), apply static forces proportional to the weights on the masses to get a final modal vector.

(2)



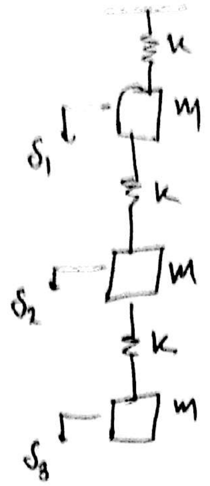
$$(\omega)_{exact} = 0.445\sqrt{\frac{k}{m}}$$

Obtain $\omega_1 \approx (\omega_R)$ by Rayleigh method.

Take $\{\delta\}$ = static deflection vector (normalized) as the trial modal vector

Compute % error.

5. For the static deflection vector,



$$\boxed{\delta_1 = \frac{3mg}{k}}$$

$$\delta_2 = \frac{2mg}{k} + \delta_1$$

$$\Rightarrow \boxed{\delta_2 = \frac{5mg}{k}}$$

$$\Rightarrow \delta_3 = \frac{mg}{k} + \delta_2$$

$$\Rightarrow \boxed{\delta_3 = \frac{6mg}{k}}$$

$$\therefore \{A_r\} = \{\delta\} = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \begin{Bmatrix} 3mg/k \\ 5mg/k \\ 6mg/k \end{Bmatrix}.$$

$$\Rightarrow \{\delta\} = \begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix} \quad \text{(normalizing } \{\delta\}\text{)}$$

from the DEOM,

$$m\ddot{x}_1 = k(x_2 - x_1) - kx_1$$
$$m\ddot{x}_2 = k(x_3 - x_2) - k(x_2 - x_1)$$
$$m\ddot{x}_3 = -k(x_3 - x_2)$$

$$\therefore [k] = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$$

and $[m] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$

FROM Rayleigh Method.

$$\therefore \omega_r^2 = \frac{\{A_r\}^T [k] \{A_r\}}{\{A_r\}^T [m] \{A_r\}}$$

$$= \frac{\{3 \quad 5 \quad 6\} \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix}}{\{3 \quad 5 \quad 6\} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix}} = \frac{14k}{70m}$$

$\Rightarrow \omega_R^2 = \dfrac{K}{5m}$

$\Rightarrow \omega_R = 0.4472\sqrt{\dfrac{K}{m}}$

$\therefore$ % error $= \dfrac{-(0.445 - 0.4472)}{0.445} \times 100$

$\Rightarrow$ $\boxed{\%\ \text{error} = 0.49}$

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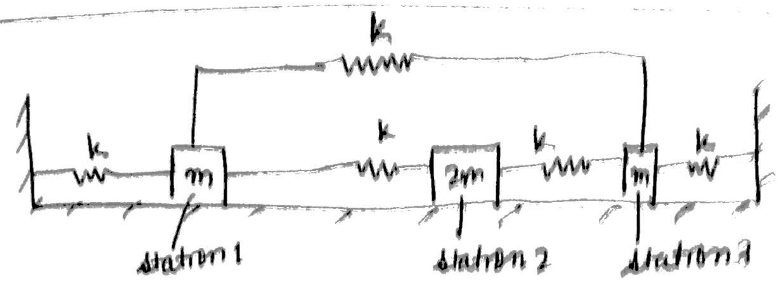
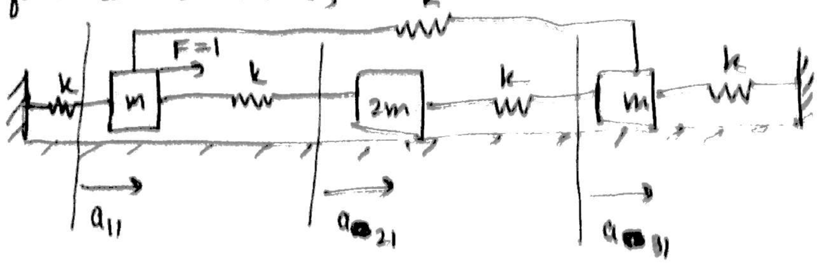
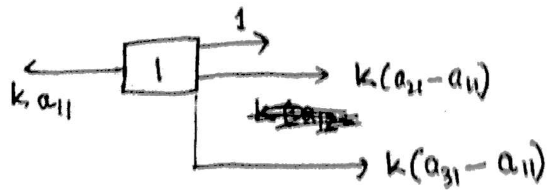
1).

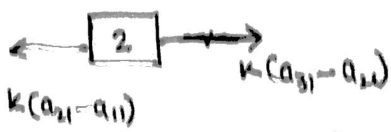


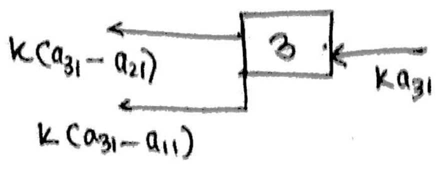
$$[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \begin{bmatrix} 5/8 & 1/2 & 3/8 \\ 1/2 & 1 & 1/2 \\ 3/8 & 1/2 & 5/8 \end{bmatrix}$$

applying force at station 1,









$\therefore ka_{11} = 1 + ka_{21} - ka_{11} + ka_{31} - ka_{11}$

$\Rightarrow 3ka_{11} = 1 + ka_{21} + ka_{31}$ — (i)

$ka_{21} - ka_{11} = ka_{31} - ka_{21}$

$\Rightarrow 2ka_{21} = ka_{31} + ka_{11}$ — (ii)

$ka_{31} - ka_{21} + ka_{31} + ka_{31} - ka_{11} = 0$

$\Rightarrow 3ka_{31} = ka_{21} + ka_{11}$ — (iii)

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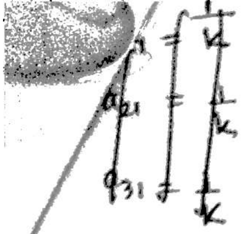
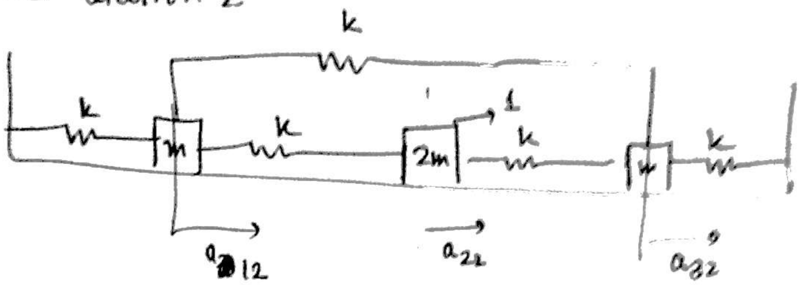


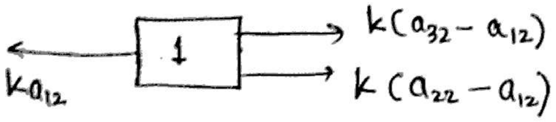
$a_{11} = \frac{5}{8k}$

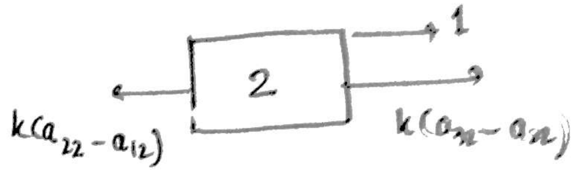
$a_{21} = \frac{1}{2k}$

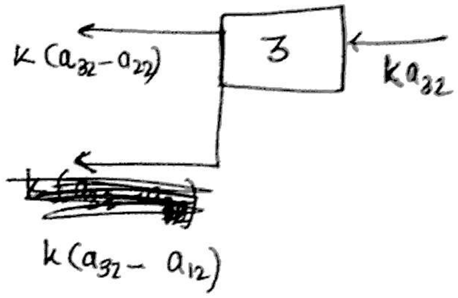
$a_{31} = \frac{3}{8k}$.

Applying force at station 2









$ka_{12} = ka_{32} - ka_{12} + ka_{22} - ka_{12}$

$\Rightarrow 3ka_{12} = ka_{32} + ka_{22}$ — (iv)

$ka_{22} - ka_{12} = 1 + ka_{32} - ka_{22}$

$\Rightarrow 2ka_{22} = 1 + ka_{32} + ka_{12}$ — (v)

$ka_{32} - ka_{22} + ka_{32} - ka_{12} + ka_{32} = 0$

$\Rightarrow 3ka_{32} = ka_{12} + ka_{22}$ — (vi)

$\therefore \quad a_{12} = \frac{1}{2k}$

$a_{22} = 1/k$

$a_{32} = \frac{1}{2k}$

Force at station 3.

$3ka_{13} = ka_{33} + ka_{23}$ — (vii)

$2ka_{23} = ka_{33} + ka_{13}$ — (viii)

$3ka_{33} = ka_{13} + ka_{23} + 1$ — (ix)

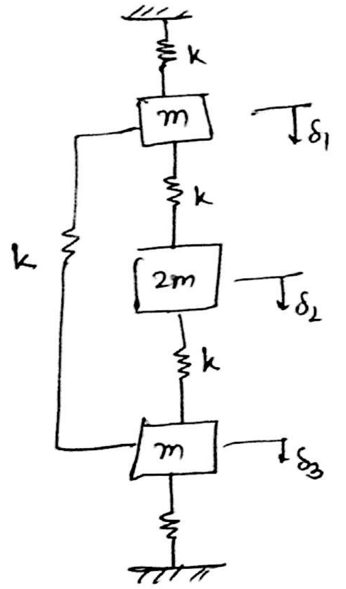
$\therefore \quad a_{13} = \frac{3}{8k}$; $a_{23} = \frac{1}{2k}$; $a_{33} = \frac{5}{8k}$

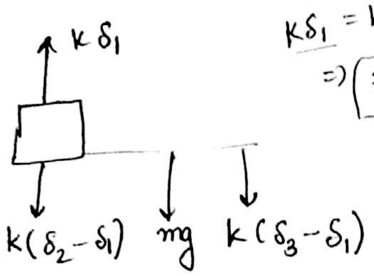
$$\therefore \quad a = \begin{bmatrix} 5/8 & 1/2 & 3/8 \\ 1/2 & 1 & 1/2 \\ 3/8 & 1/2 & 5/8 \end{bmatrix} \frac{1}{k}$$

$$\therefore \quad [k] = [a]^{-1}$$

$$\Rightarrow \boxed{[k] = \begin{bmatrix} 3k & -k & -k \\ -k & 2k & -k \\ -k & -k & 3k \end{bmatrix}}$$

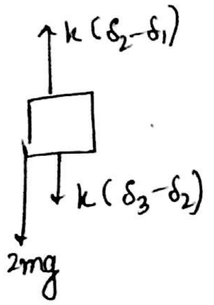
To obtain static deflection matrix





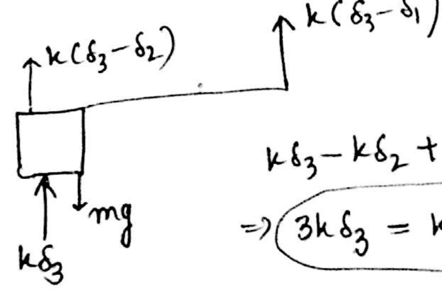
$$k\delta_1 = k\delta_2 - k\delta_1 + k\delta_3 - k\delta_1 + mg$$

$$\Rightarrow \boxed{3k\delta_1 = k\delta_2 + k\delta_3 + mg}$$



$$k\delta_2 - k\delta_1 = k\delta_3 - k\delta_2 + 2mg$$

$$\Rightarrow \boxed{2k\delta_2 = k\delta_1 + k\delta_3 + 2mg}$$



$$k\delta_3 - k\delta_2 + k\delta_3 - k\delta_1 + k\delta_3 = mg$$

$$\Rightarrow \boxed{3k\delta_3 = k\delta_2 + k\delta_1 + mg}$$

$$\therefore \delta_1 = \frac{2mg}{k} \qquad \delta_2 = \frac{3mg}{k} \qquad \delta_3 = \frac{2mg}{k}$$

$$A_1 = [\delta] = \begin{Bmatrix} 2 \\ 3 \\ 2 \end{Bmatrix} \qquad m = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix}$$

$$\omega_r^2 = \frac{\{2 \;\; 3 \;\; 2\} \begin{bmatrix} 3k & -k & -k \\ -k & 2k & -k \\ -k & -k & 3k \end{bmatrix} \begin{Bmatrix} 2 \\ 3 \\ 2 \end{Bmatrix}}{\{2 \;\; 3 \;\; 2\} \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} 2 \\ 3 \\ 2 \end{Bmatrix}} = \frac{10k}{26m}$$

$$\boxed{\therefore \; \omega_r = 0.6202 \sqrt{\frac{k}{m}}}$$

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