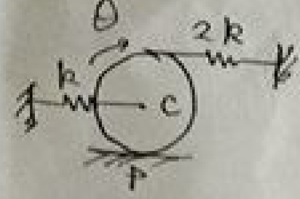
Brief Solutions

(1)



$$T = \frac{1}{2} I_P \dot{\theta}^2, \quad I_P = \frac{3}{2} m r^2$$

$$U = \frac{1}{2} k (r\theta)^2 + \frac{1}{2} \cdot 2k (2r\theta)^2 = \frac{9}{2} k r^2 \theta^2$$

Let $\theta = \theta_m \sin \omega_n t$

$\therefore U_{max} = 9/2\, k r^2 \theta_m^2, \quad T_{max} = \frac{3}{4} m r^2 \theta_m^2 \omega_n^2$

By Rayleigh's method, $T_{max} = U_{max} \Rightarrow \omega_n = \sqrt{\frac{6k}{m}}$

(2) Mass of shaft $= 0.1764$ kg

$I_{shaft} = 3.1752 \times 10^{-6}$ kgm^2

Mass of disc $= 0.9924$ kg

$I_{disc} = 40.19 \times 10^{-4}$ kgm^2

$k_t = \frac{GJ}{l} = 814.3$ N-m/rad

$\omega_n = \sqrt{\frac{k_t}{I_{disc}}} = 450.125$ rad/s

(If shaft inertia is considered, $\omega_n = \sqrt{\frac{k_t}{I_{disc} + \frac{1}{3} I_{shaft}}} = 450.06$ rad/s)

(3) $k_t = \frac{GJ}{l} = 55.56$ N-m/rad

(a) $\delta = \ln \frac{14.4}{1.2} = \ln \frac{1.2}{0.1} = 2.4849$

(b) $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = \frac{\delta}{\sqrt{\delta^2 + 4\pi^2}} = 0.368$

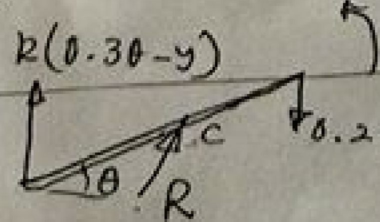
(c) $\omega_n = \sqrt{\frac{k_t}{I_d}} = 9.6229$ rad/s

(d) $\omega_d = \omega_n \sqrt{1-\zeta^2} = 8.9484$ rad/s

(e) $c_t = 2\zeta \sqrt{I_d k_t} = 4.2489$ N-m/(rad/s)

PTO

(4)



$\curvearrowleft \theta, \dot{\theta}, \ddot{\theta}$ +ive

$\Sigma M_c = 0 \Rightarrow 0.25\ddot{\theta} + 2.4\dot{\theta} + 63\theta = 2.1 \sin 10t$ Nm

Compare with $I_C \ddot{\theta} + c_t \dot{\theta} + k_t \theta = M_0 \sin \omega_f t$

$I_c = 0.25$ kgm^2, $c_t = 2.4$ N-m-s/rad

$k_t = 63$ N-m/rad, $M_0 = 2.1$ N-m

$\omega_n = \sqrt{\frac{63}{0.25}} = 15.87$ rad/s, $r = \frac{10}{\omega_n} = 0.63$

$\zeta = \frac{c_t}{2\sqrt{I_c k_t}} = 0.3024$. Reqd. amplitude $= \frac{M_0/k_t}{\ldots}$ = 0.0442 .

(5) (a) DEOM:-

$$5.5\ddot{x} + 40\dot{x} + 1000x = 0$$

or,

$$\ddot{x} + 7.273\dot{x} + 181.82x = 0$$

$\xi = 0.27 < 1$, $\omega_n = 13.48$ rad/s, $\omega_d = 12.98$ rad/s

$$x = X_0 e^{-3.64t} \sin(12.98t + \phi)$$

$x(0) = 50$ mm, $\dot{x}(0) = 0 \Rightarrow X_0 = 51.93$ mm, $\phi = 74.34^\circ = 1.2974$ rad

Hence, $x(t) = 51.93 e^{-3.64t} \sin(12.98t + 1.2974)$ mm

(b) $\frac{dx}{dt} = 0 \Rightarrow t = \frac{n\pi}{12.98}$. Here $n=1 \Rightarrow t = 0.2419$ s

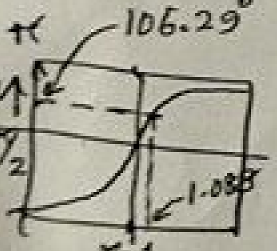
$x|_{t=0.2419s} = -20.72$ mm.

$\therefore$ Reqd. distance $= 50 + 20.72 = 70.72$ mm

(c) $x = 0 \Rightarrow t = 0.142$ s

(6) $\omega_f = 3 \times 2\pi = 18.85$ rad/s, $F_0 = 80$ N, $k = 30 \times 10^3$ N/m

$\omega_n = 17.32$ rad/s, $r = 1.088$, $\xi = 0.289$



Reqd. amplitude $= \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = 4.07$ mm

$$\psi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right) = 180^\circ - 73.71^\circ = 106.29^\circ \text{ (1.85 rad)}$$