

# PROBLEMS (1)

1.9 Find the equivalent torsional spring constant of the system shown in Fig. 1.61. Assume that  $k_1, k_2, k_3,$  and  $k_4$  are torsional and  $k_5$  and  $k_6$  are linear spring constants.

1.12 The tripod shown in Fig. 1.64 is used for mounting an electronic instrument that finds the distance between two points in space. The legs of the tripod are located symmetrically about the mid-vertical axis, each leg making an angle  $\alpha$  with the vertical. If each leg has a length  $l$  and axial stiffness  $k$ , find the equivalent spring stiffness of the tripod in the vertical direction.

1.10 A machine of mass  $m = 500$  kg is mounted on a simply supported steel beam of length  $l = 2$  m having a rectangular cross section (depth = 0.1 m, width = 1.2 m) and Young's modulus  $E = 2.06 \times 10^{11}$  N/m<sup>2</sup>. To reduce the vertical deflection of the beam, a spring of stiffness  $k$  is attached at the mid-span, as shown in Fig. 1.62. Determine the value of  $k$  needed to reduce the deflection of the beam by

- 25 percent of its original value
- 50 percent of its original value
- 75 percent of its original value.

Assume that the mass of the beam is negligible.

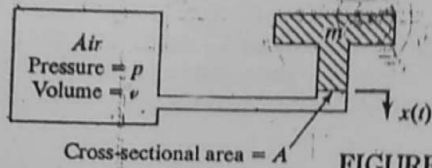


FIGURE 1.67

1.16 Figure 1.67 shows an air spring. This type of spring is generally used for obtaining very low natural frequencies while maintaining zero deflection under static loads. Find the spring constant of this air spring by assuming that the pressure  $p$  and volume  $v$  change adiabatically when the mass  $m$  moves.

Hint:

$pv^\gamma = \text{constant}$  for an adiabatic process, where  $\gamma$  is the ratio of specific heats. For air,  $\gamma = 1.4$ .

1.19 The force ( $F$ )–deflection ( $x$ ) relationship of a nonlinear spring is given by

$$F = ax + bx^3$$

where  $a$  and  $b$  are constants. Find the equivalent linear spring constant when the deflection is 0.01 m with  $a = 20,000$  N/m and  $b = 40 \times 10^6$  N/m<sup>3</sup>.

1.30 In Fig. 1.76 find the equivalent mass of the rocker arm assembly with respect to the  $x$  coordinate.

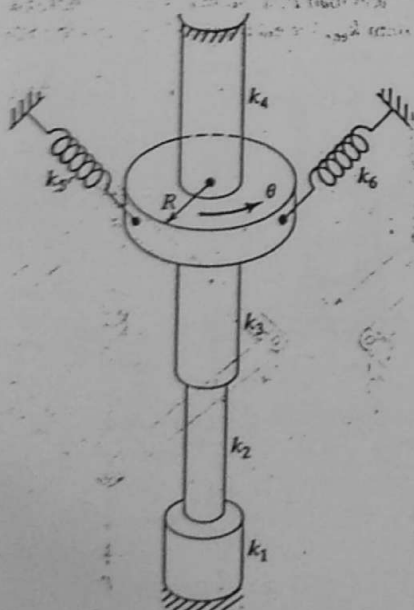


FIGURE 1.61

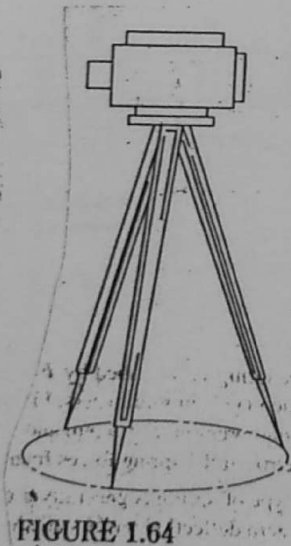


FIGURE 1.64

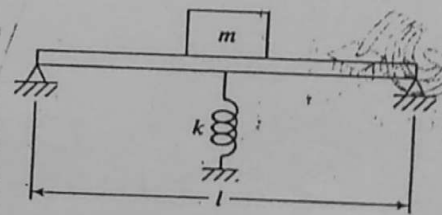


FIGURE 1.62

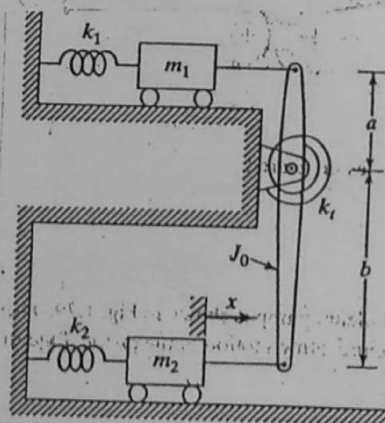


FIGURE 1.76 Rocker arm assembly.

PTO

1.31 Find the equivalent mass moment of inertia of the gear train shown in Fig. 1.77 with reference to the driving shaft. In Fig. 1.77,  $J_i$  and  $n_i$  denote the mass moment of inertia and the number of teeth, respectively, of gear  $i$ ,  $i = 1, 2, \dots, 2N$ .

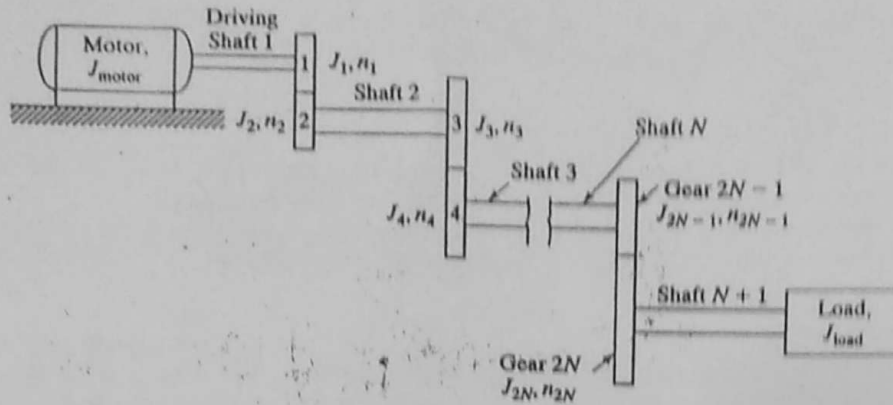


FIGURE 1.77

1.32 Two masses, having mass moments of inertia  $J_1$  and  $J_2$ , are placed on rotating rigid shafts that are connected by gears, as shown in Fig. 1.78. If the number of teeth on gears 1 and 2 are  $n_1$  and  $n_2$ , respectively, find the equivalent mass moment of inertia corresponding to  $\theta_1$ .

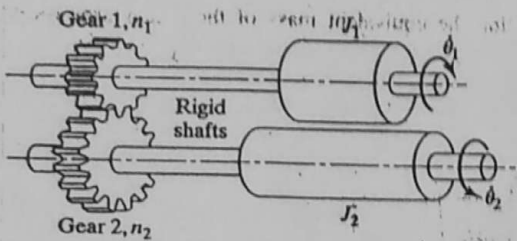


FIGURE 1.78 Rotational masses on geared shafts.

1.33 A simplified model of a petroleum pump is shown in Fig. 1.79, where the rotary motion of the crank is converted to the reciprocating motion of the piston. Find the equivalent mass,  $m_{eq}$ , of the system at location A.

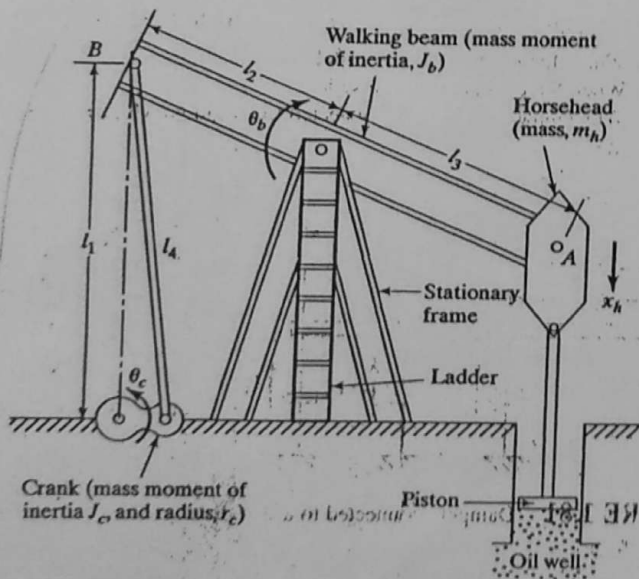


FIGURE 1.79

1.34 Find the equivalent mass of the system shown in Fig. 1.80.

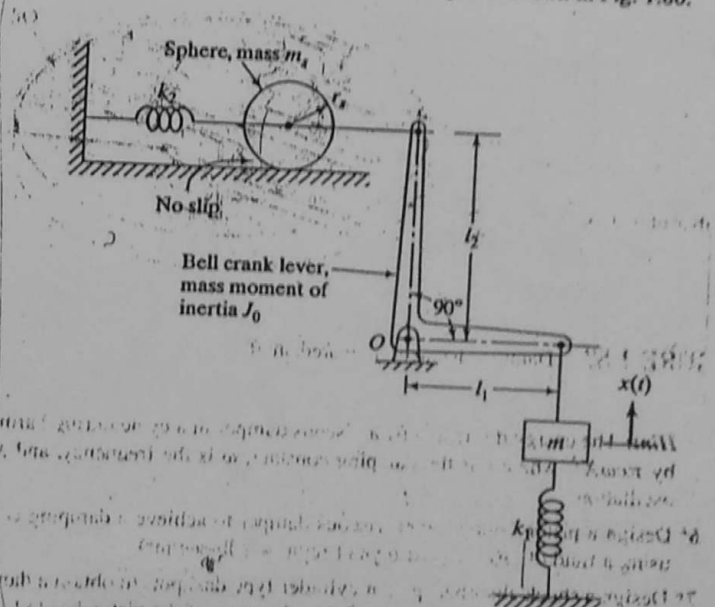


FIGURE 1.80

PTO

# PROBLEMS

An automobile having a mass of 2,000 kg deflects its suspension springs 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible.

- 9 Find the natural frequency of vibration of a spring-mass system arranged on an inclined plane, as shown in Fig. 2.41.
- 2.10 A loaded mine cart, weighing 5,000 lb, is being lifted by a frictionless pulley and a wire rope, as shown in Fig. 2.42. Find the natural frequency of vibration of the cart in the given position.
- 2.12 Find the natural frequency of the system shown in Fig. 2.44 with and without the springs  $k_1$  and  $k_2$  in the middle of the elastic beam.
- 2.15 A rigid block of mass  $M$  is mounted on four elastic supports, as shown in Fig. 2.47. A mass  $m$  drops from a height  $h$  and adheres to the rigid block without rebounding. If the spring constant of each elastic support is  $k$ , find the natural frequency of vibration of the system (a) without the mass  $m$ , and (b) with the mass  $m$ . Also find the resulting motion of the system in case (b).
- 2.18 A heavy machine weighing 9,810 N is being lowered vertically down by a winch at a uniform velocity of 2 m/s. The steel cable supporting the machine has a diameter of 0.01 m. The winch is suddenly stopped when the steel cable's length is 20 m. Find the period and amplitude of the ensuing vibration of the machine.



FIGURE 2.41

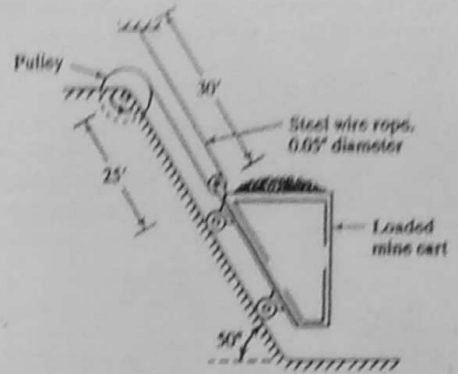


FIGURE 2.42

- 2.39 A pick-and-place robot arm, shown in Fig. 2.69, carries an object weighing 10 lb. Find the natural frequency of the robot arm in the axial direction for the following data:  $l_1 = 12$  in.,  $l_2 = 10$  in.,  $l_3 = 8$  in.;  $E_1 = E_2 = E_3 = 10^7$  psi;  $D_1 = 2$  in.,  $D_2 = 1.5$  in.,  $D_3 = 1$  in.;  $d_1 = 1.75$  in.,  $d_2 = 1.25$  in.,  $d_3 = 0.75$  in.
- 2.64 One of the blades of an electric fan is removed (as shown by dotted lines in Fig. 2.81). The steel shaft  $AB$ , on which the blades are mounted, is equivalent to a uniform shaft of diameter 1 in. and length 6 in. Each blade can be modeled as a uniform slender rod of weight 2 lb and length 12 in. Determine the natural frequency of vibration of the remaining three blades about the  $y$ -axis.

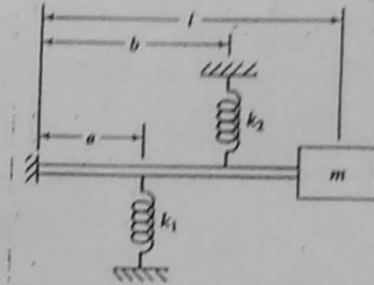


FIGURE 2.44

- 2.84 A simple pendulum is found to vibrate at a frequency of 0.5 Hz in a vacuum and 0.45 Hz in a viscous fluid medium. Find the damping constant, assuming the mass of the bob of the pendulum is 1 kg.
- 2.85 The ratio of successive amplitudes of a viscously damped single degree of freedom system is found to be 18:1. Determine the ratio of successive amplitudes if the amount of damping is (a) doubled, and (b) halved.
- 2.86 Assuming that the phase angle is zero, show that the response  $x(t)$  of an underdamped single degree of freedom system reaches a maximum value when

$$\sin \omega_d t = \sqrt{1 - \zeta^2}$$

and a minimum value when

$$\sin \omega_d t = -\sqrt{1 - \zeta^2}$$

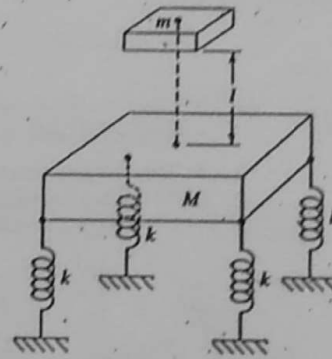
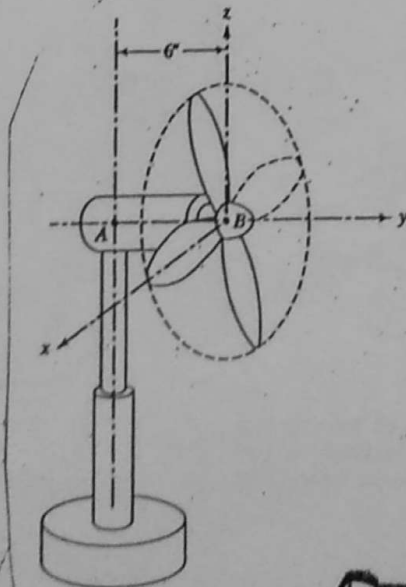


FIGURE 2.47

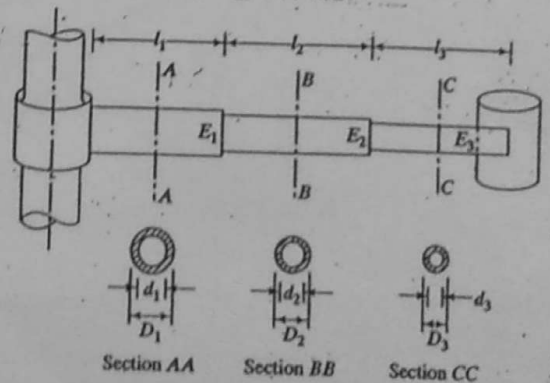


FIGURE 2.69

PTO

# PROBLEMS (2)

- 2.87 Derive an expression for the time at which the response of a critically damped system will attain its maximum value. Also find the expression for the maximum response.
- 2.88 A shock absorber is to be designed to limit its overshoot to 15 percent of its initial displacement when released. Find the damping ratio  $\zeta_0$  required. What will be the overshoot if  $\zeta$  is made equal to (a)  $\frac{3}{4}\zeta_0$ , and (b)  $\frac{5}{4}\zeta_0$ ?
- 2.89 The free vibration responses of an electric motor of weight 500 N mounted on different types of foundations are shown in Figs. 2.91 (a) and (b). Identify the following in each case: (i) the nature of damping provided by the foundation, (ii) the spring constant and damping coefficient of the foundation, and (iii) the undamped and damped natural frequencies of the electric motor.
- 2.90 For a spring-mass-damper system,  $m = 50$  kg and  $k = 5,000$  N/m. Find the following: (a) critical damping constant  $c_c$ , (b) damped natural frequency when  $c = c_c/2$ , and (c) logarithmic decrement.
- 2.91 A railroad car of mass 2,000 kg traveling at a velocity  $v = 10$  m/s is stopped at the end of the tracks by a spring-damper system, as shown in Fig. 2.92. If the stiffness of the spring is  $k = 40$  N/mm and the damping constant is  $c = 20$  N-s/mm, determine (a) the maximum displacement of the car after engaging the springs and damper and (b) the time taken to reach the maximum displacement.

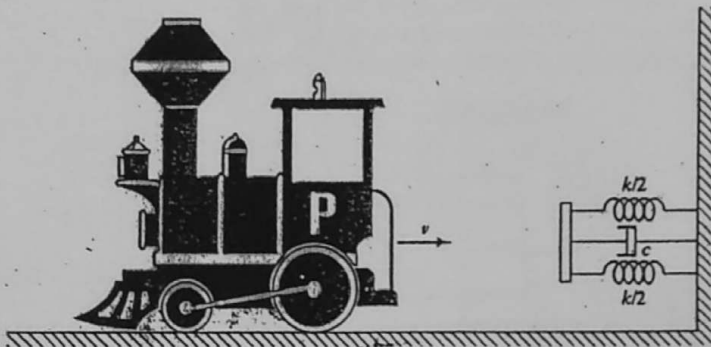
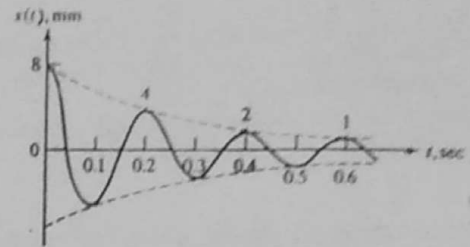
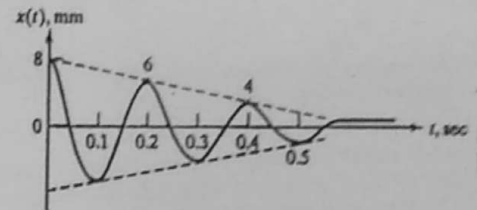


FIGURE 2.92



(a)



(b)

FIGURE 2.91

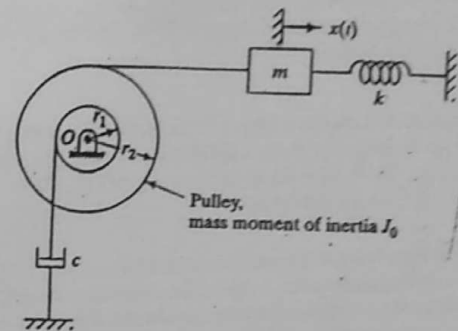


FIGURE 2.97

- 2.106 The system shown in Fig. 2.97 has a natural frequency of 5 Hz for the following data:  $m = 10$  kg,  $J_0 = 5$  kg-m<sup>2</sup>,  $r_1 = 10$  cm,  $r_2 = 25$  cm. When the system is disturbed by giving it an initial displacement, the amplitude of free vibration is reduced by 80 percent in 10 cycles. Determine the values of  $k$  and  $c$ .

- 2.121 The mass of a spring-mass system vibrates on a dry surface inclined at  $30^\circ$  to the horizontal as shown in Fig. 2.99.

- Derive the equation of motion.
- Find the response of the system for the following data:  
 $m = 20$  kg,  $k = 1000$  N/m,  $\mu = 0.1$ ;  $x_0 = 0.1$  m,  $\dot{x}_0 = 5$  m/s.

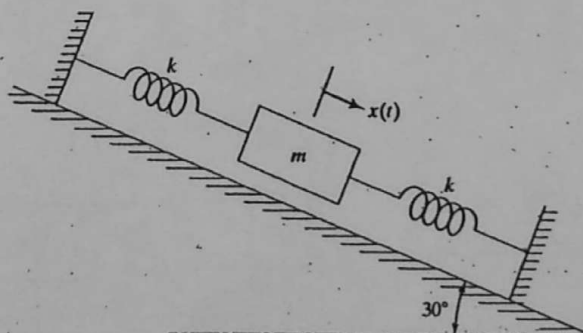


FIGURE 2.99

- 2.112 A mass of 20 kg slides back and forth on a dry surface due to the action of a spring having a stiffness of 10 N/mm. After four complete cycles, the amplitude has been found to be 100 mm. What is the average coefficient of friction between the two surfaces if the original amplitude was 150 mm? How much time has elapsed during the four cycles?