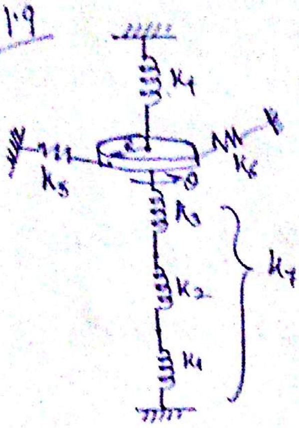
Dynamics of Machines

Tutorial Sheet-1

1.9



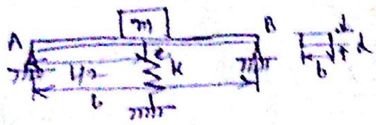
$$U = \frac{1}{2}k_4\theta^2 + \frac{1}{2}k_7\theta^2 + \frac{1}{2}k_5(\theta R)^2 + \frac{1}{2}k_6(\theta R)^2 = \frac{1}{2}k_{eq}\theta^2$$

$$\frac{1}{k_4} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \Rightarrow k_4 = \frac{k_1k_2k_3}{k_1k_2 + k_2k_3 + k_3k_1}$$

$$\therefore \boxed{k_{eq} = \frac{k_1k_2k_3}{k_1k_2 + k_2k_3 + k_3k_1} + k_7 + (k_5 + k_6)R^2} \text{ (Ans.)}$$

1.10

$m = 500$ kg, $l = 2$ m, $d = 0.1$ m, $b = 1.2$ m, $E = 2.06 \times 10^{11}$ N/m²



For simply supported beam, $k_b = \frac{48EI}{l^3}$ $\left[\because \delta_{mid} = \frac{Pl^3}{48EI}\right]$ $= \frac{48Ebd^3}{12l^3} = \frac{4Ebd^3}{l^3}$

$I = \frac{bd^3}{12}$

without k, $\delta_e = \frac{mg}{k_b} = \frac{mgl^3}{4Ebd^3}$

With k, $k_{eq} = k_b + k = \left(\frac{4Ebd^3}{l^3} + k\right) \Rightarrow \delta_e' = \frac{mg}{\left(\frac{4Ebd^3}{l^3} + k\right)} = \frac{mgl^3}{4Ebd^3 + kl^3}$

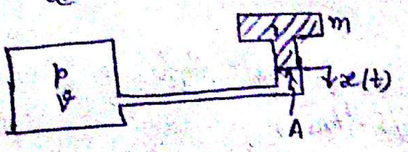
$$\therefore \frac{\delta_e'}{\delta_e} = \frac{4Ebd^3}{4Ebd^3 + kl^3} = \frac{1}{1 + \frac{kl^3}{4Ebd^3}} = \frac{1}{1 + 8.0906 \times 10^{-6} k}$$

(a) $\frac{\delta_e'}{\delta_e} = 0.25 \Rightarrow$ $k = 3.708 \times 10^8$ N/m. (Ans.)

(b) $\frac{\delta_e'}{\delta_e} = 0.5 \Rightarrow$ $k = 1.236 \times 10^8$ N/m. (Ans.)

(c) $\frac{\delta_e'}{\delta_e} = 0.75 \Rightarrow$ $k = 0.412 \times 10^8$ N/m. (Ans.)

1.16



pv^γ = constant, $\gamma = 1.4$ for air

pv^γ = constant

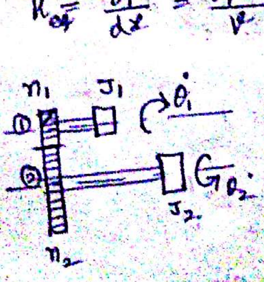
$\Rightarrow v^\gamma dp + \gamma p v^{\gamma-1} dv = 0 \Rightarrow dp = \frac{-\gamma p}{v} dv$

$dv = -A\,dx$ (As m lowers by amount dx)

$\therefore$ Force generated $= dF = A\,dp = A\cdot\left(\frac{-\gamma p}{v}\right)dv = A\left(\frac{-\gamma p}{v}\right)\cdot(-A\,dx) = \frac{\gamma p A^2}{v}dx$

$\therefore k_{eq} = \frac{dF}{dx} = \frac{\gamma p A^2}{v}$. (Ans.)

1.32



$$T = \frac{1}{2}J_1\dot\theta_1^2 + \frac{1}{2}J_2\dot\theta_2^2 = \frac{1}{2}J_{eq}\dot\theta_1^2$$

$$n_1\dot\theta_1 = n_2\dot\theta_2 \Rightarrow \dot\theta_2 = \frac{n_1\dot\theta_1}{n_2}$$

$$\therefore \boxed{J_{eq} = J_1 + J_2\frac{\dot\theta_2^2}{\dot\theta_1^2} = J_1 + \left(\frac{n_1}{n_2}\right)^2 J_2} \text{ (Ans.)}$$

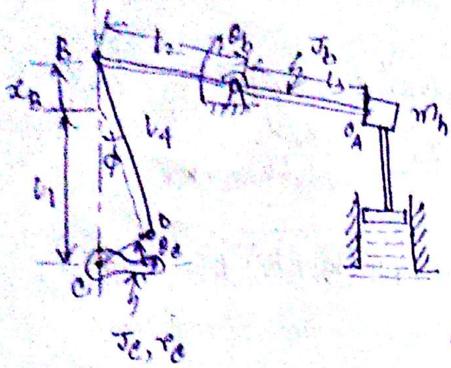
1.31 let, motor's angular velocity $= \dot\theta$

$\therefore \omega_1 = \dot\theta$, $\omega_2 = \dot\theta\left(\frac{n_1}{n_2}\right)$, $\omega_3 = \omega_2\left(\frac{n_3}{n_4}\right) = \dot\theta\left(\frac{n_1n_3}{n_2n_4}\right)$, ..., $\omega_{N+1} = \dot\theta\left(\frac{n_1n_3 \cdots n_{2N-1}}{n_2n_4 \cdots n_{2N}}\right)$

$\therefore T = \frac{1}{2}(J_{Motor} + J_1)\omega_1^2 + \frac{1}{2}(J_2 + J_3)\omega_2^2 + \cdots + \frac{1}{2}(J_{2N} + J_{load})\omega_{N+1}^2 = \frac{1}{2}J_{eq}\dot\theta^2$

$$\therefore \boxed{J_{eq} = (J_{Motor} + J_1) + (J_2 + J_3)\cdot\left(\frac{n_1}{n_2}\right)^2 + (J_4 + J_5)\cdot\left(\frac{n_1n_3}{n_2n_4}\right)^2 + \cdots + (J_{2N} + J_{load})\left(\frac{n_1n_3 \cdots n_{2N-1}}{n_2n_4 \cdots n_{2N}}\right)^2}$$

1.33



$\theta_b = \frac{x_h}{l_3}$

$\therefore x_B = l_2\theta_b = x_h \frac{l_2}{l_3}$

From ΔBCD,

$x_B + l_1 = r_c \sin\theta_c + l_4 \cos\phi$

$\Rightarrow x_B = r_c \sin\theta_c + (l_4\cos\phi - l_1)$

As $l_4 >> r_c$, both θ_c & ϕ are small

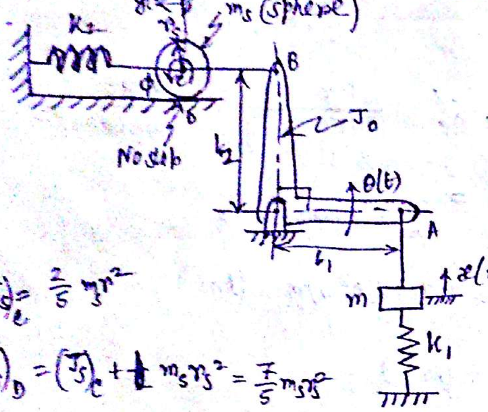
$\Rightarrow \cos\phi \approx 1,\ l_4 \approx l_1,\ \sin\theta_c \approx \theta_c$

$\therefore x_B = r_c\theta_c \Rightarrow \theta_c = \frac{x_B}{r_c} = \frac{x_h}{r_c}\cdot\frac{l_2}{l_3}$

$\therefore T = \frac{1}{2} J_c \dot\theta_c^2 + \frac{1}{2} J_b \dot\theta_b^2 + \frac{1}{2} m_h \dot x_h^2 = \frac{1}{2} m_{eq} \dot x_h^2$

$$\Rightarrow \boxed{m_{eq} = m_h + \frac{J_b}{l_3^2} + J_c\left(\frac{l_2}{r_c l_3}\right)^2} \quad (\text{Ans.})$$

1.34



$\theta = \frac{x}{l_1},\ y = l_2\theta = \frac{x l_2}{l_1}$

$\phi = \frac{y}{r_s} = \frac{x l_2}{r_s l_1}$

$T = \frac{1}{2} m\dot x^2 + \frac{1}{2} J_0 \dot\theta^2 + \frac{1}{2} (J_s)_0 \dot\phi^2$

$= \frac{1}{2} m_{eq} \dot x^2$

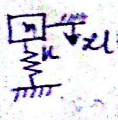
$(J_s)_c = \frac{2}{5} m_s r_s^2$

$(J_s)_0 = (J_s)_c + m_s r_s^2 = \frac{7}{5} m_s r_s^2$

$$\therefore \boxed{m_{eq} = m + \frac{J_0}{l_1^2} + \frac{7}{5} m_s \left(\frac{l_2^2}{l_1^2}\right)} \quad (\text{Ans.})$$

Tutorial Sheet-2

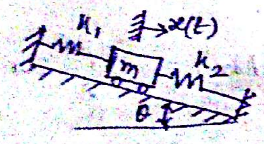
2.8 $m = 2000\,kg,\ \delta_{st} = 0.02\,m$

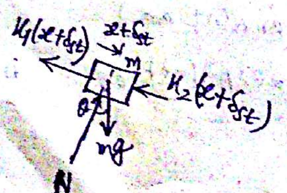


$m\ddot x + kx = 0$ $\quad mg = k\delta_{st} \Rightarrow \delta_{st} = \frac{mg}{k} \Rightarrow \frac{k}{m} = \frac{g}{\delta_{st}}$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_{st}}} = 22.1472$ rad/s. (Ans.)

2.9





$m\ddot x = mg\sin\theta - k_1(x+\delta_{st}) - k_2(x+\delta_{st})$

$\Rightarrow m\ddot x + (k_1+k_2)x = 0$

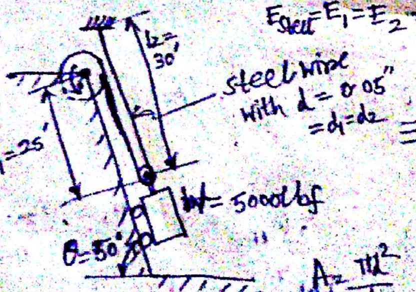
$mg\sin\theta = k_1\delta_{st} + k_2\delta_{st}$

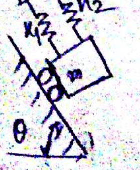
$\Rightarrow \delta_{st} = \left(\frac{mg\sin\theta}{k_1+k_2}\right)$

$\therefore \omega_n = \sqrt{\frac{k_1+k_2}{m}}$ (Ans.)

2.10

$E_{steel} = E_1 = E_2 = 30\times10^6$ psi





$k_1 = \frac{E_1A_1}{l_1},\ k_2 = \frac{E_2A_2}{l}$

$m\ddot x + (k_1+k_2)x = 0$

$\therefore \omega_n = \sqrt{\frac{k_1+k_2}{m}}$

$A = \frac{\pi d^2}{4}$

$m = \frac{W}{g} = 12.94$ lbs

$g = 32.2\,ft/s^2 = 386.4\,inch/s^2$

$= \sqrt{\frac{E_1A_1}{ml_1} + \frac{E_2A_2}{ml_2}} = \sqrt{\frac{\pi E d^2}{4m}\left(\frac{1}{l_1}+\frac{1}{l_2}\right)}$

$= 5.2744$ ~~0.0583~~ rad/s. (Ans.)

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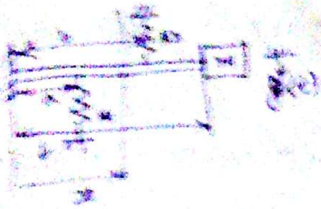
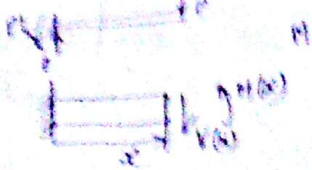
2.4





$M(x) = -EI\frac{d^2y}{dx^2} = -PL + Px$

$-EI\frac{d^2y}{dx^2} = -P(L-x)$

$-EI\frac{dy}{dx} = \frac{P(L-x)^2}{2} + A_1$

$\Rightarrow -EIy = -\frac{P}{6}(L-x)^3 + A_1x + A_2$

$y|_{x=0} = 0,\ \frac{dy}{dx}\Big|_{x=0} = 0 \qquad A_1 = -\frac{PL^2}{2},\ A_2 = \frac{PL^3}{6}$

$-EIy = -\frac{PL^3}{6} + \frac{PL^2x}{2} - \frac{PLx^2}{2} + \frac{Px^3}{6} - \frac{PL^2x}{2} + \frac{PL^3}{6} = \frac{Px^2}{6}(x-3L)$

$\Rightarrow y = \frac{Px^2}{6EI}(3L-x)$

$\therefore y_a = \frac{Pa^2(3L-a)}{6EI},\ y_b = \frac{Pb^2(3L-b)}{6EI},\ y_L = \frac{PL^3}{3EI}$ [here, $P = mg$]

$U_{beam} = \int_0^L \frac{M(x)^2}{2EI}dx = \frac{P^2L^3}{6EI} = \frac{9E^2I^2y_L^2}{6EI\,L^6}\cdot L^3 = \frac{3EI\,y_L^2}{2L^3} = \frac{1}{2}k_{beam}y_L^2$

$\therefore k_{beam} = \frac{P}{y_L} = \frac{3EI}{L^3}$

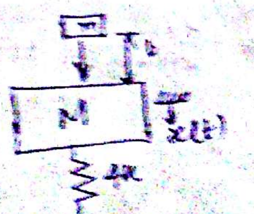
$\therefore \omega_n$ without springs $= \sqrt{\frac{k_{beam}}{m}} = \sqrt{\frac{3EI}{mL^3}}$ (Ans.)

with springs, $U = \frac{1}{2}k_1y_a^2 + \frac{1}{2}k_2y_b^2 + \frac{1}{2}k_{beam}y_L^2 = \frac{1}{2}k_{eq}y_L^2$

$\therefore \omega_n$ with springs $= \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1\left(\frac{y_a}{y_L}\right)^2 + k_2\left(\frac{y_b}{y_L}\right)^2 + k_{beam}}{m}}$

$= \sqrt{\frac{k_1}{m}\left\{\frac{a^2(3L-a)}{2L^3}\right\}^2 + \frac{k_2}{m}\left\{\frac{b^2(3L-b)}{2L^3}\right\}^2 + \frac{3EI}{mL^3}}$ (Ans.)

2.5



(a) without mass m, $\omega_n = \sqrt{\frac{4k}{M}}$ (Ans.)

(b) m drops from height l and adheres block M.

$v_0\big|_m = \sqrt{2gl}$, $x_0\big|_{(m+M)} = \frac{mg}{4k}$

$(m+M)\,v_0\big|_{(m+M)} = m\,v_0\big|_m + 0 \Rightarrow v_0\big|_{(m+M)} = \left(\frac{m}{m+M}\right)\sqrt{2gl}$

$(m+M)\ddot{x} + 4kx = 0$ $\qquad \omega_n = \sqrt{\frac{4k}{m+M}}$

$\therefore x(t) = A_0\sin(\omega_nt + \phi_0) \Rightarrow \dot{x}(t) = A_0\omega_n\cos(\omega_nt + \phi_0)$

$x(0) = A_0\sin\phi_0 = \frac{mg}{4k}$

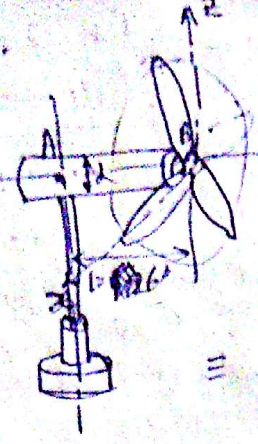
$\dot{x}(0) = A_0\omega_n\cos\phi_0 = \left(\frac{m}{m+M}\right)\sqrt{2gl} \Rightarrow A_0\cos\phi_0 = \frac{m\sqrt{2gl(m+M)}}{(m+M)\sqrt{4k}} = \frac{m\sqrt{gl}}{\sqrt{2k(m+M)}}$

$\therefore A_0 = \sqrt{\frac{m^2g^2}{16k^2} + \frac{m^2gl}{2k(m+M)}}$, $\phi_0 = \tan^{-1}\left(\frac{mg\sqrt{2k(m+M)}}{4k\,m\sqrt{gl}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{g(m+M)}}{2\sqrt{2kl}}\right)$

$x(t) = \sqrt{\frac{m^2g^2}{16k^2} + \frac{m^2gl}{2k(m+M)}}\ \sin\left[\omega_nt + \tan^{-1}\left\{\frac{\sqrt{g(m+M)}}{2\sqrt{2kl}}\right\}\right]$ (Ans.)

2.64.



For AB shaft, $l_s = 6$ inch, $d_s = 1$ inch

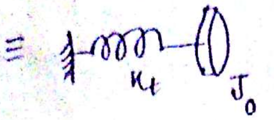
For each blade, $l_b = 12$ inch, $W_b = 2$ lbf

for steel, $G = 12\times10^6$ psi

$\Rightarrow m_b = \frac{W_b}{g}$

$g = 32.2$ ft/s² $= 386.4$ inch/s²

$$k_t = \frac{G J_s}{l_s} = \frac{\pi d_s^4 G}{32 l_s}$$

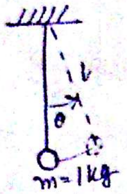


J_0 = Mass moment of inertia of all three blades about y axis

$$= 3 \times \frac{1}{3} m_b l_b^2 = m_b l_b^2$$

$$\therefore \omega_n = \sqrt{\frac{k_t}{J_0}} = \sqrt{\frac{\pi d_s^4 G g}{32 l_s W_b l_b^2}} = 513.2598 \text{ rad/s . (Ans.)}$$

2.84. For a simple pendulum, $\omega_n = \sqrt{\frac{g}{l}} = 0.5\text{ Hz} = 3.1416$ rad/s



$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 0.45\text{ Hz} = 2.5133 \text{ rad/s}$$

$$\therefore \zeta = \sqrt{\frac{\omega_n^2-\omega_d^2}{\omega_n^2}} = 0.4359 \qquad l = \frac{g}{\omega_n^2} = 0.994\text{ m}$$

$$ml^2\ddot{\theta} + c_t\dot{\theta} + mgl\theta = 0$$

$$c_{ct} = 2(ml^2)\omega_n = 6.2075 \text{ N-m-s/rad}$$

$$\zeta = \frac{c_t}{c_{ct}} \Rightarrow c_t = \zeta c_{ct} = 2.7059 \text{ N-m-s/rad. (Ans).}$$

2.85.

$$\frac{x(t)}{x(t+\tau)} = 18 \qquad \therefore \delta = \ln(18) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 2.8904$$

$$\therefore \zeta = \sqrt{\frac{\delta^2}{\delta^2+4\pi^2}} = 0.4179$$

(a) For $\zeta_{new} = 2\zeta$, $\delta = \frac{2\pi\zeta_{new}}{\sqrt{1-\zeta_{new}^2}} = 9.5663 \Rightarrow \frac{x(t)}{x(t+\tau)} = 14276.1731$. (Ans.)

(b) For $\zeta_{new} = \zeta/2$, $\delta = \frac{2\pi\zeta_{new}}{\sqrt{1-\zeta_{new}^2}} = 1.3426 \Rightarrow \frac{x(t)}{x(t+\tau)} = 3.8292$ (Ans.)

2.86. $x(t) = A_0 e^{-\zeta\omega_n t}\sin\omega_d t$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\Rightarrow \dot{x}(t) = -A_0\zeta\omega_n e^{-\zeta\omega_n t}\sin\omega_d t + A_0\omega_d e^{-\zeta\omega_n t}\cos\omega_d t$$
$$= A_0 e^{-\zeta\omega_n t}(-\zeta\omega_n\sin\omega_d t + \omega_d\cos\omega_d t)$$

For $x(t_m) = x_m$, $\dot{x}(t_m) = 0$

$$\Rightarrow -\zeta\omega_n\sin\omega_d t_m + \omega_d\cos\omega_d t_m = 0 \Rightarrow \tan(\omega_d t_m) = \frac{\omega_d}{\zeta\omega_n} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$\therefore$ (A) $\sin(\omega_d t_m) = \sqrt{1-\zeta^2}$, & $\cos(\omega_d t_m) = \zeta$, or, (B) $\sin(\omega_d t_m) = -\sqrt{1-\zeta^2}$, & $\cos(\omega_d t_m) = -\zeta$

$$\ddot{x}(t) = A_0 e^{-\zeta\omega_n t}\left[\zeta^2\omega_n^2\sin\omega_d t - 2\zeta\omega_n\omega_d\cos\omega_d t - \omega_d^2\sin\omega_d t\right]$$

For case (A), $\ddot{x}(t_m) = A_0 e^{-\zeta\omega_n t_m}\left\{\zeta^2\omega_n^2\sqrt{1-\zeta^2} - 2\zeta^2\omega_n^2\sqrt{1-\zeta^2} - \omega_n^2(1-\zeta^2)^{3/2}\right\}$

$$= -A_0 e^{-\zeta\omega_n t_m}\,\omega_n^2\sqrt{1-\zeta^2}\;(\ldots) < 0$$

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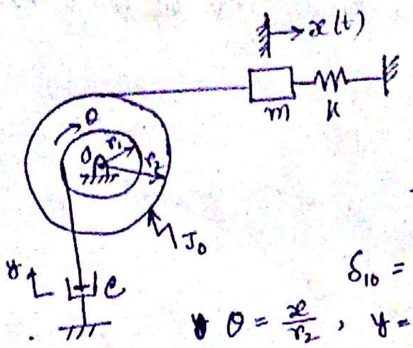
$\therefore$ For $x(t_m)$ to be x_{max}, $\sin(\omega_d t_m) = \sqrt{1-\xi^2}$. (Proved.)

For case (B), $\ddot{x}(t_m) = A_0 e^{-\xi\omega_n t_m}\left\{-\xi^2\omega_n^2\sqrt{1-\xi^2} + 2\xi^2\omega_n^2\sqrt{1-\xi^2} + \omega_n^2(1-\xi^2)^{3/2}\right\}$

$= A_0 e^{-\xi\omega_n t_m}\,\omega_n^2\sqrt{1-\xi^2} > 0$

$\therefore$ For $x(t_m)$ to be x_{min}, $\sin(\omega_d t_m) = -\sqrt{1-\xi^2}$. (Proved.)

2.106



$m = 10\,kg$, $J_0 = 5\,kg\text{-}m^2$, $r_1 = 0.1\,m$, $r_2 = 0.25\,m$,

$\omega_n = 5\,Hz = 31.4159\,rad/s$.

$\frac{x(t+10\tau)}{x(t)} = (1-0.8) = 0.2$

$\delta_{10} = \frac{1}{10}\ln\left\{\frac{x(t)}{x(t+10\tau)}\right\} = \frac{1}{10}\ln\left(\frac{1}{0.2}\right) = 0.1609$

$\theta = \frac{x}{r_2}$, $y = r_1\theta = \frac{x r_1}{r_2}$

$U = \frac{1}{2}kx^2 = \frac{1}{2}k_{eq}x^2 \Rightarrow k_{eq} = k$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}m_{eq}\dot{x}^2 \Rightarrow m_{eq} = m + \frac{J_0}{r_2^2} = 90\,kg$

$\Delta E = \frac{1}{2}c\dot{y}^2 = \frac{1}{2}c_{eq}\dot{x}^2 \Rightarrow c_{eq} = c\left(\frac{r_1}{r_2}\right)^2 = 0.16\,c$

$\therefore m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = 0$

$\Rightarrow \left(m + \frac{J_0}{r_2^2}\right)\ddot{x} + c\left(\frac{r_1}{r_2}\right)^2\dot{x} + kx = 0$

$\zeta = \sqrt{\frac{\delta^2}{4\pi^2+\delta^2}} = 0.0256 = \frac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}}$

$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = 31.4159 \Rightarrow k_{eq} = 88.8263\,kN/m$

$\therefore c_{eq} = 144.7645\,N\text{-}s/m$

$\therefore k = 88.8263\,kN/m$. (Ans.)

$c = 904.7779\,N\text{-}s/m$. (Ans.)

2.18 $m = \frac{9810\,N}{9.81\,m/s^2} = 1000\,kg$, $E = 210\times10^9\,Pa$, $L = 20\,m$, $d = 0.01\,m$

$\therefore k = \frac{EA}{L} = \frac{\pi d^2 E}{4L} = 824.6681\,kN/m$

$\omega_n = \sqrt{\frac{k}{m}} = 28.717\,rad/s$ $\therefore \tau_n = \frac{2\pi}{\omega_n} = 0.2187$ sec. (Ans.)

$\therefore x(t) = X_0\sin(\omega_n t + \phi_0) \Rightarrow \dot{x}(t) = X_0\omega_n\cos(\omega_n t + \phi_0)$

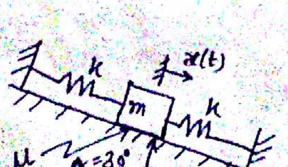
$x_0 = x(0) = 0$, $\dot{x}_0 = \dot{x}(0) = 2\,m/s$

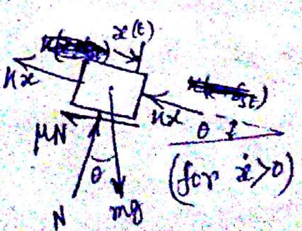
$\Rightarrow X_0\sin\phi_0 = 0$ $\Rightarrow X_0\omega_n\cos\phi_0 = 2$

$\Rightarrow \phi_0 = 0$ $\Rightarrow X_0 = \frac{2}{\omega_n} = 0.0696\,m$. (Ans.)

2.121

$m = 20\,kg$, $k = 1000\,N/m$, $\mu = 0.1$, $x_0 = 0.1\,m$, $\dot{x}_0 = 5\,m/s$





$N = mg\cos\theta$

$2k\delta_{st} + \mu mg\cos\theta = mg\sin\theta$

(i) For $\dot{x}>0$, $m\ddot{x} = -2kx + mg\sin\theta - \mu mg\cos\theta \Rightarrow m\ddot{x} + 2kx = -\mu mg\cos\theta + mg\sin\theta$

For $\dot{x}<0$, $m\ddot{x} = -2kx + mg\sin\theta + \mu mg\cos\theta \Rightarrow m\ddot{x} + 2kx = \mu mg\cos\theta + mg\sin\theta$ (Ans.)

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$\therefore\ m\ddot{x} + kx - mg\sin\theta + \mu mg\cos\theta\ \mathrm{sgn}(\dot{x}) = 0$. (Ans.)

For $\dot{x} > 0$, $x(t) = A_1\cos\omega_n t + B_1\sin\omega_n t - \frac{\mu mg\cos\theta}{k} + \frac{mg\sin\theta}{k}$ ... (1)

For $\dot{x} < 0$, $x(t) = A_2\cos\omega_n t + B_2\sin\omega_n t + \frac{\mu mg\cos\theta}{k} + \frac{mg\sin\theta}{k}$ ... (2)

$\omega_n = \sqrt{\frac{k}{m}} = 7.0711$ rad/s

For first half cycle $(\dot{x} > 0)$; $x_0 = 0.1\,m$, $\dot{x}_0 = 5\,m/s$

Using these A_1 & B_1 can be solved from (1).

For second half cycle, solve (2) using the end conditions of the first half cycle as the initial conditions.

2.112 $m = 20\,kg$, $k = 10\,N/mm = 10000\,N/m$, $x_0 = 180\,mm = 0.18\,m$, $x_4 = 100\,mm = 0.1\,m$

$x_0 - x_1 = \frac{4\mu_k mg}{k}$ (for each cycle)

$\Rightarrow x_0 - x_4 = 4 \times \frac{4\mu_k mg}{k}$

$\therefore 4 \times \frac{4\mu_k mg}{k} = x_0 - x_4$

$\Rightarrow \mu_k = 0.1593$. (Ans.)

$\omega_n = \sqrt{\frac{k}{m}}$

$\therefore$ Time taken $= 4\tau_n = 4 \times \frac{2\pi}{\omega_n} = 8\pi\sqrt{\frac{m}{k}} = 1.124$ sec. (Ans.)