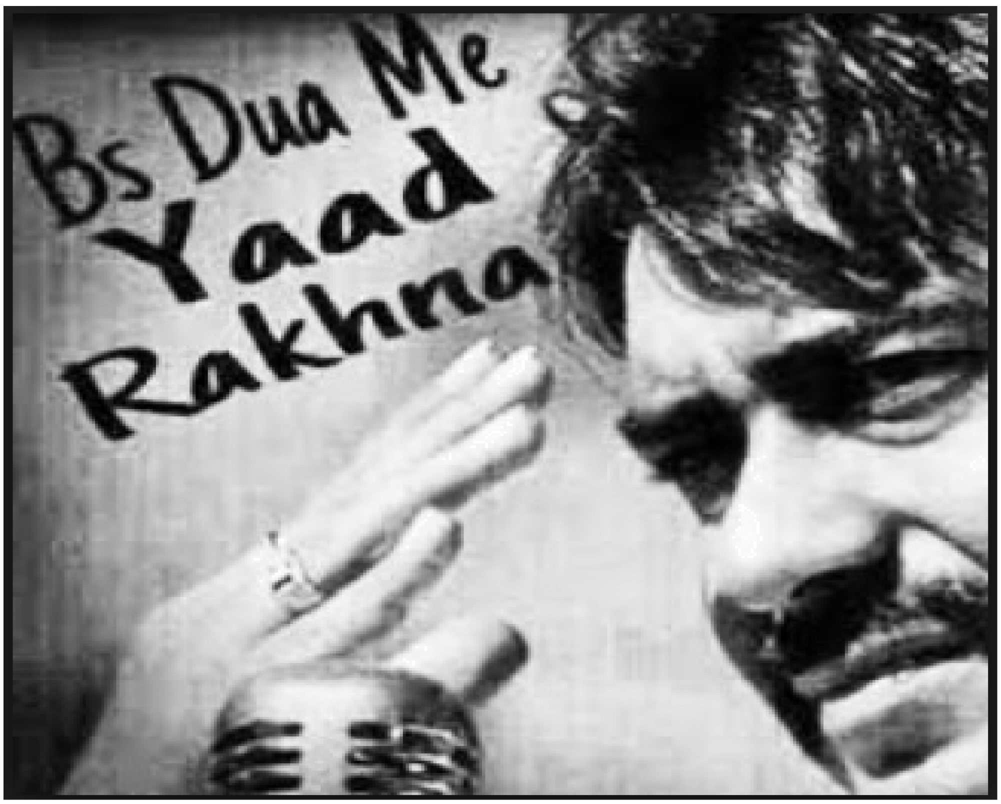
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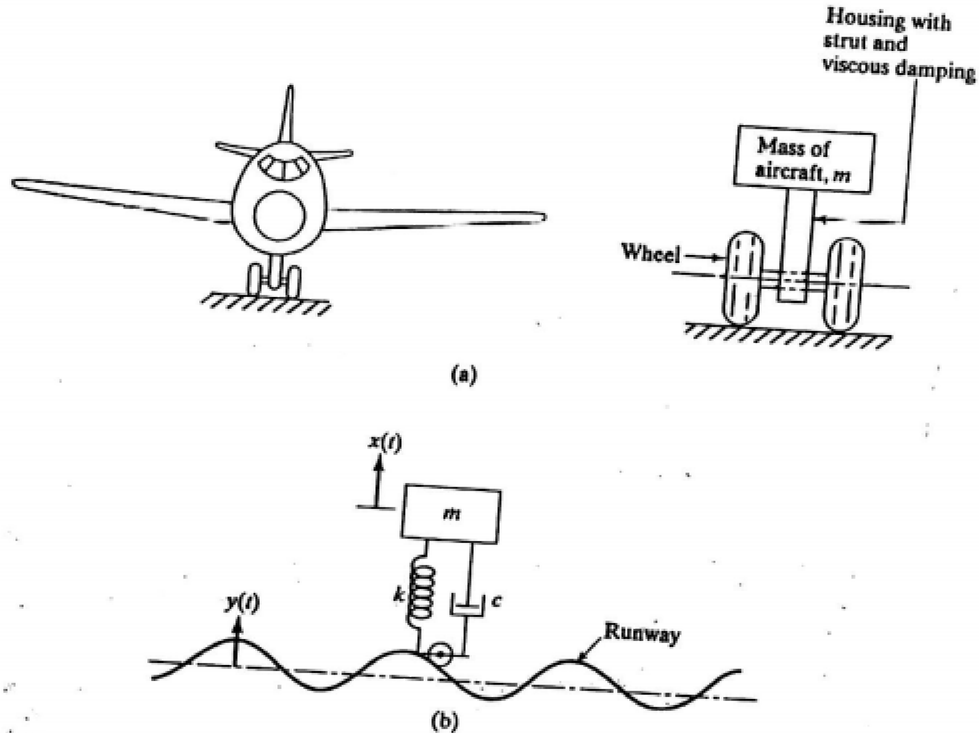
3.35 The landing gear of an airplane can be idealized as the spring-mass-damper system shown in Fig. 3.45. If the runway surface is described $y(t) = y_0 \cos \omega t$, determine the values of k and c that limit the amplitude of vibration of the airplane (x) to 0.1 m. Assume $m = 2000$ kg, $y_0 = 0.2$ m, and $\omega = 157.08$ rad/s.



3.44 Amplitude of vibration under base excitation:

$$X = Y \left\{ \frac{\sqrt{k^2 + (c\,\omega)^2}}{\left[\left(k - m\,\omega^2 \right)^2 + (c\,\omega)^2 \right]^{\frac{1}{2}}} \right\}$$

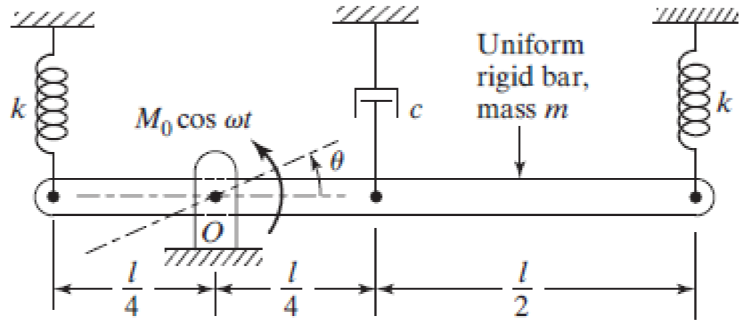
$$= \frac{(0.2)\sqrt{k^2 + c^2\,(157.08)^2}}{\left[\left\{ k - 2000\,(157.08)^2 \right\}^2 + c^2\,(157.08)^2 \right]^{\frac{1}{2}}} = 0.1 \text{ m} \qquad (1)$$

Let $k = 5\,(10^6)$ N/m. Then Eq. (1) gives:

$$\frac{\sqrt{25\,(10^{12}) + 2.4674\,(10^4)\,c^2}}{\sqrt{1966.7717\,(10^{12}) + 2.4674\,(10^4)\,c^2}} = 0.5$$

i.e., $1.85055\,(10^4)\,c^2 = 466.6929\,(10^{12})$ i.e., $c = 158805.0$ N–s/m

3.46 Derive the equation of motion and find the steady-state response of the system shown in Fig. 3.54 for rotational motion about the hinge O for the following data: $k = 5000$ N/m, $l = 1$ m, $c = 1000$ N-s/m, $m = 10$ kg, $M_0 = 100$ N-m, $\omega = 1000$ rpm.



6

Equation of motion:

$$I_0\, \ddot{\theta} + \left(k\, \frac{\ell}{4}\, \theta \right) \frac{\ell}{4} + \left(c\, \frac{\ell}{4}\, \dot{\theta} \right) \frac{\ell}{4} + \left(k\, \frac{3\,\ell}{4}\, \theta \right) \frac{3\,\ell}{4} = M_0 \cos \omega\, t$$

$$\text{or} \quad I_0\, \ddot{\theta} + c\, \frac{\ell^2}{16}\, \dot{\theta} + \frac{5}{8}\, k\, \ell^2\, \theta = M_0 \cos \omega\, t$$

$$\text{where } I_0 = \frac{m\, \ell^2}{12} + m \left(\frac{\ell}{4}\right)^2 = \frac{7}{48}\, m\, \ell^2 = \frac{7}{48}\, (10)\, (1^2) = 1.4583 \text{ kg--m}^2$$

$$\frac{c\, \ell^2}{16} = \frac{(1000)\, (1^2)}{16} = 62.5 \text{ N--m--s/rad}$$

$$\frac{5}{8}\, k\, \ell^2 = \frac{5}{8}\, (5000)\, (1^2) = 3125.0 \text{ N--m/rad}$$

$$\omega = \frac{1000\, (2\, \pi)}{60} = 104.72 \text{ rad/sec}$$

Equation of motion becomes:

$$1.4583\, \ddot{\theta} + 62.5\, \dot{\theta} + 3125.0\, \theta = 100 \cos 104.72\, t$$

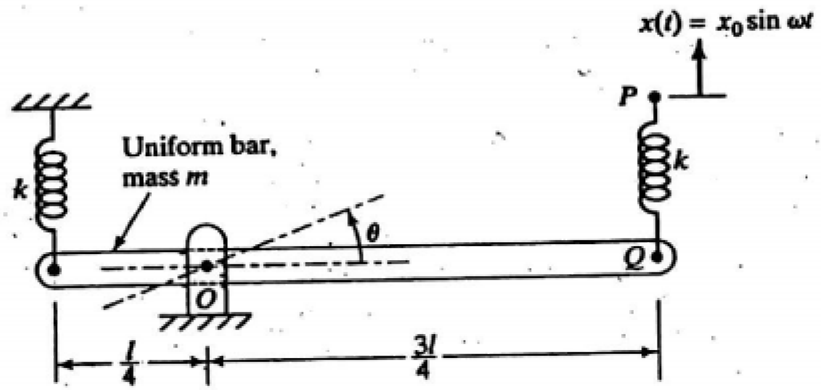
Steady state response is given by Eq. (3.28):

$$\theta_p(t) = \Theta \cos (\omega\, t - \phi) = \Theta \cos (104.72\, t - \phi)\text{.sp}$$

$$\text{where} \quad \Theta = \frac{100}{\left[\left\{ 3125.0 - 1.4583\, (104.72^2) \right\}^2 + \left\{ 62.5\, (104.72) \right\}^2 \right]^{\frac{1}{2}}} = 0.006927 \text{ rad}$$

$$\text{and} \quad \phi = \tan^{-1} \left(\frac{62.5\, (104.72)}{3125.0 - 1.4583\, (104.72^2)} \right) = -\,0.4705 \text{ rad} = -\,26.9606^\circ$$

3.47 A uniform bar of mass m is pivoted at point O and supported at the ends by two springs, as shown in Fig. 3.52. End P of spring PQ is subjected to a sinusoidal displacement, $x(t) = x_0 \sin \omega t$. Find the steady-state angular displacement of the bar when $l = 1$ m, $k = 1000$ N/m, $m = 10$ kg, $x_0 = 1$ cm, and $\omega = 10$ rad/s.

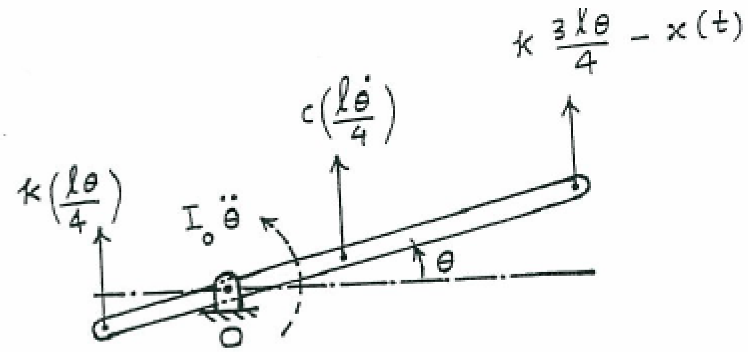


Equation of motion:

$$I_0 \, \ddot{\theta} = - k \, \frac{\ell \, \theta}{4} \, (\frac{\ell}{4}) - c \, \frac{\ell}{4} \, \dot{\theta} \, (\frac{\ell}{4}) - k \left(\frac{3 \, \ell}{4} \, \theta - x(t) \right) \frac{3 \, \ell}{4}$$

$$\text{i.e.,} \quad I_0 \, \ddot{\theta} + \frac{1}{16} \, c \, \ell^2 \, \dot{\theta} + \frac{5}{8} \, k \, \ell^2 = \frac{3}{4} \, k \, \ell \, x(t) = \frac{3}{4} \, k \, \ell \, x_0 \, \sin \omega \, t \tag{1}$$

$$\text{where} \quad I_0 = \frac{1}{12} \, m \, \ell^2 + m \left(\frac{\ell}{4} \right)^2 = \frac{7}{48} \, m \, \ell^2 = \frac{7}{48} \, (10) \, (1^2) = 1.4583 \text{ kg—m}^2 \tag{2}$$



Using given data, Eq. (1) can be expressed as

$$1.4583 \, \ddot{\theta} + \frac{1}{16} \, (500) \, (1^2) \, \dot{\theta} + \frac{5}{8} \, (1000) \, (1^2) \, \theta = \frac{3}{4} \, (1000) \, (1) \, (0.01) \, \sin 10$$

$$\text{i.e.,} \quad 1.4583 \, \ddot{\theta} + 31.25 \, \dot{\theta} + 625.0 \, \theta = 7.5 \, \sin 10 \, t$$

Steady state angular displacement of the bar is given by Eq. (3.28) with:

$$\Theta = \frac{7.5}{\left\{ \left[625.0 - 1.4583 \, (10^2) \right]^2 + 31.25^2 \, (10^2) \right\}^{\frac{1}{2}}} = 0.01311 \text{ rad}$$

$$\phi = \tan^{-1} \left(\frac{31.25 \, (10)}{625.0 - 1.4583 \, (10^2)} \right) = 0.5779 \text{ rad}$$

$$\therefore \, \theta(t) = \Theta \, \sin \, (\omega \, t - \phi) = 0.01311 \, \sin \, (10 \, t - 0.5779) \text{ rad}$$

3.68 A centrifugal pump, weighing 600 N and operating at 1000 rpm, is mounted on six springs of stiffness 6000 N/m each. Find the maximum permissible unbalance in order to limit the steady-state deflection to 5 mm peak-to-peak.

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$m = (600/9.81)$ N , $\quad \omega = 2\pi(1000)/60 = 104.72$ rad/sec

$k = 6\,(6000) = 36{,}000$ N/m

$\omega_n = \sqrt{k/m} = \sqrt{36000/\left(\frac{600}{9.81}\right)} = 24.2611$ rad/sec

$r = \omega/\omega_n = 104.72/24.2611 = 4.3164$, $\quad r^2 = 18.6311$

$$X = \frac{F_0}{k\,|r^2-1|} = \frac{m_0\, e\, \omega^2}{k\,|r^2-1|}$$ where m_0 = unbalanced mass and e = eccentricity

i.e., $$2.5 \times 10^{-3} = \frac{m_0\, e\, (104.72)^2}{36000\,|17.6311|}$$

i.e., $m_0 e = 0.1447$ kg-m

$\therefore$ Unbalance $= W_0\, e = m_0 g e = 0.1447\,(9.81) = 1.4195$ N-m

3.7 A spring-mass system consists of a mass weighing 100 N and a spring with a stiffness of 2000 N/m. The mass is subjected to resonance by a harmonic force $F(t) = 25 \cos \omega t$ N. Find the amplitude of the forced motion at the end of (a) $\frac{1}{4}$ cycle, (b) $2\frac{1}{2}$ cycles, and (c) $5\frac{3}{4}$ cycles.

3.7

$\delta_{st} = \frac{F_0}{k} = \frac{25}{2000} = 0.0125$ m

steady state solution at resonance $= x(t) = \frac{\delta_{st} \cdot \omega_n t}{2} \sin \omega_n t$

$= 0.00625\ \omega_n t \ \sin \omega_n t$ m

(a) At end of $\frac{1}{4}$ cycle, $\omega_n t = \frac{\pi}{2}$ and $x(t) = 0.00625\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} = 0.009817$ m

(b) At end of $2\frac{1}{2}$ cycles, $\omega_n t = 5\pi$ and $x(t) = 0.00625\,(5\pi) \sin 5\pi = 0$

(c) At end of $5\frac{3}{4}$ cycles, $\omega_n t = 11\frac{1}{2}\pi$ and

$x(t) = 0.00625\left(\frac{23}{2}\pi\right) \sin \frac{23}{2}\pi = -0.2258$ m

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3.8 A mass m is suspended from a spring of stiffness 4000 N/m and is subjected to a harmonic force having an amplitude of 100 N and a frequency of 5 Hz. The amplitude of the forced motion of the mass is observed to be 20 mm. Find the value of m.

3.8

$\delta_{st} = \frac{F_0}{k} = \frac{100}{4000} = 0.025$ m.

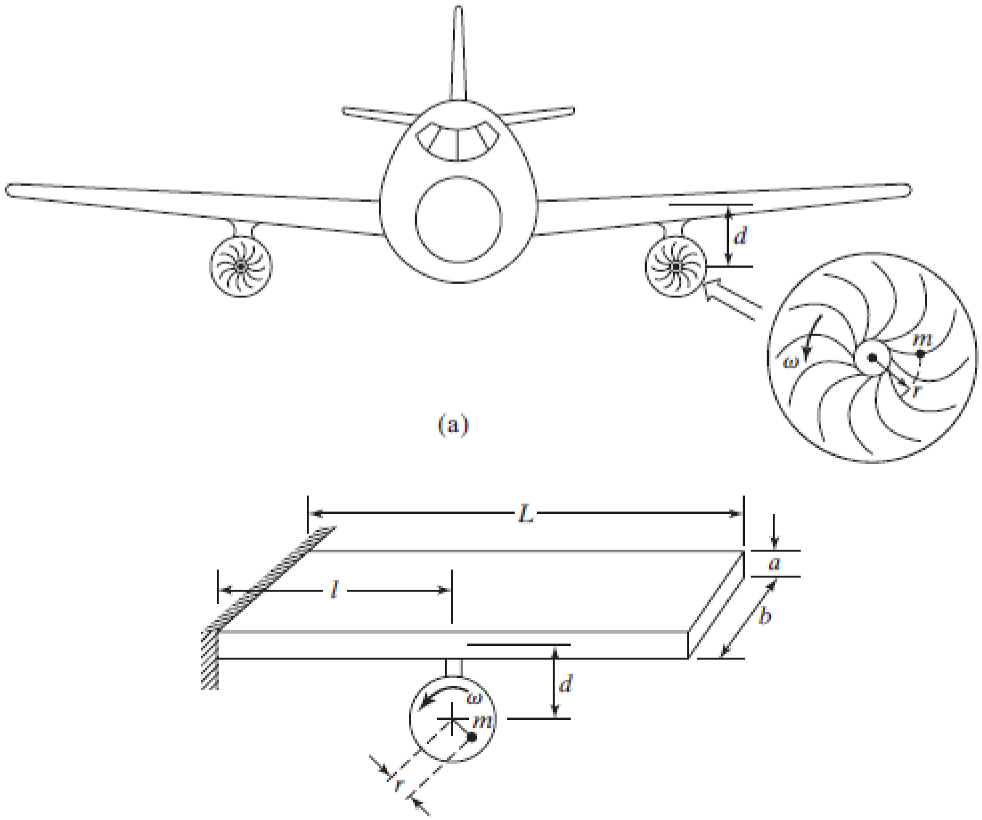
$X = \delta_{st} \cdot \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$, $\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right| = \frac{\delta_{st}}{X} = \frac{0.025}{20 \times 10^{-3}} = 1.25$

$\frac{\omega}{\omega_n} = \sqrt{1.25 + 1} = 1.5$

$\omega_n = \omega / 1.5 = 5(2\pi)/1.5 = 20.944$ rad/sec

$m = k/\omega_n^2 = 4000/(20.944)^2 = 9.1189$ kg

3.16 An aircraft engine has a rotating unbalanced mass m at radius r. If the wing can be modeled as a cantilever beam of uniform cross section $a \times b$, as shown in Fig. 3.39(b), determine the maximum deflection of the engine at a speed of N rpm. Assume damping and effect of the wing between the engine and the free end to be negligible.



(a)

(b)

3.16 Equivalent stiffness of wing (beam) at location of engine:

$$k = \frac{\text{force}}{\text{deflection}} = \frac{3\,E\,I}{\ell^3} = \frac{3\,E\,(\frac{1}{12}\,b\,a^3)}{\ell^3} = \frac{E\,b\,a^3}{4\,\ell^3}$$

Magnitude of unbalanced force: $= m\,r\,\omega^2 = m\,r \left(\frac{2\,\pi\,N}{60}\right)^2 = \frac{m\,r\,\pi^2\,N^2}{900}$

Equivalent mass of wing at location of engine: $M = \frac{33}{140}\,m_w = \frac{33}{140}\,(a\,b\,\ell\,\rho)$

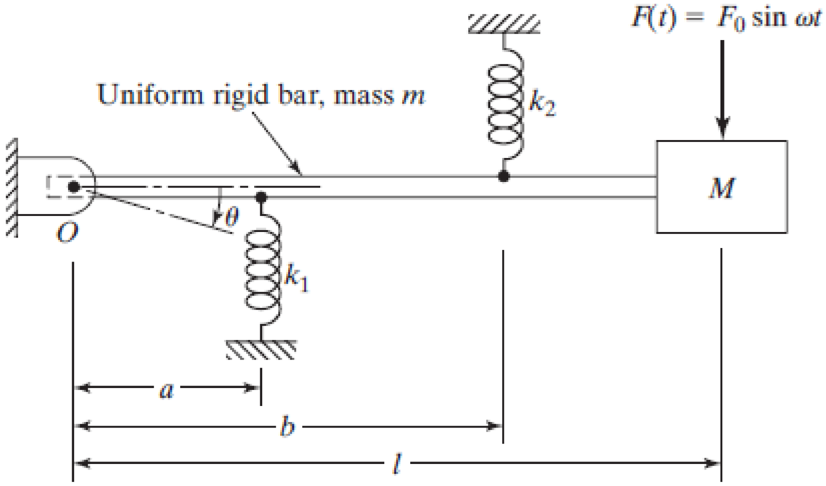
Equation of motion: $M\,\ddot{x} + k\,x = m\,r\,\omega^2 \sin \omega\,t$

Maximum steady state displacement of wing at location of engine:

$$X = \left| \frac{m\,r\,\omega^2}{k - M\,\omega^2} \right| = \left| \frac{\left(\frac{m\,r\,\pi^2\,N^2}{900}\right)}{\left\{\frac{E\,b\,a^3}{4\,\ell^3} - \frac{33}{140}\,a\,b\,\ell\,\rho \left(\frac{2\,\pi\,N}{60}\right)^2\right\}} \right|$$

$$= \left| \frac{m\,r\,\ell^3\,N^2}{22.7973\,E\,b\,a^3 - 0.2357\,\rho\,a\,b\,\ell^4\,N^2} \right|$$

3.24 Derive the equation of motion and find the steady-state response of the system shown in Fig. 3.44 for rotational motion about the hinge O for the following data: $k_1 = k_2 = 5000$ N/m, $a = 0.25$ m, $b = 0.5$ m, $l = 1$ m, $M = 50$ kg, $m = 10$ kg, $F_0 = 500$ N, $\omega = 1000$ rpm.



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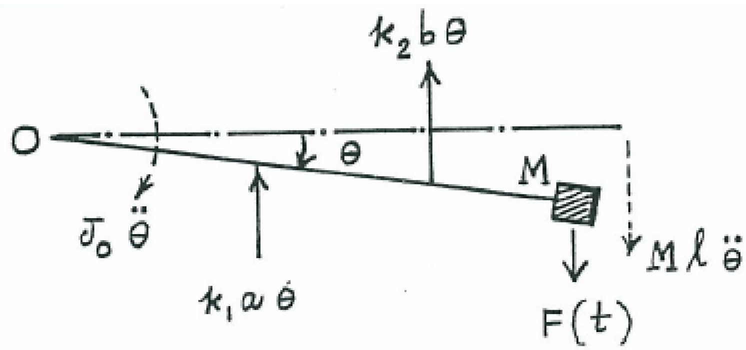


Equation of motion for rotational motion about the hinge O:

$$(J_0 + M\,\ell^2)\,\ddot{\theta} + (k_1\,a^2 + k_2\,b^2)\,\theta = F(t)\,\ell = F_0\,\ell\,\sin\omega t \qquad (1)$$

Steady state response (using Eqs. (3.3) and (3.6)):

$$\theta_p(t) = \Theta \sin \omega t \qquad (2)$$

$$\text{where} \quad \Theta = \frac{F_0\,\ell}{(k_1\,a^2 + k_2\,b^2) - (J_0 + M\,\ell^2)\,\omega^2} \qquad (3)$$

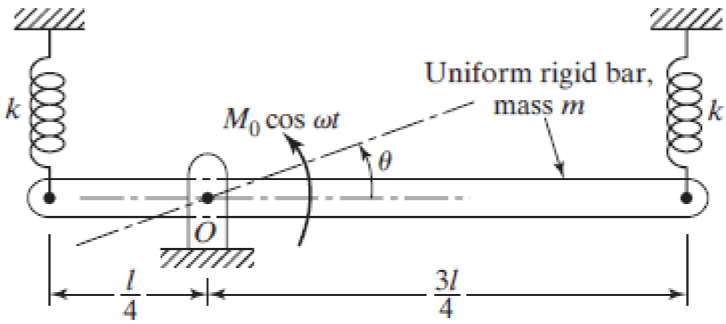
$$\text{and} \quad J_0 = \frac{m\,\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}\,m\,\ell^2 \qquad (4)$$

For given data, $J_0 = \frac{1}{3}\,(10)\,(1^2) = 3.3333$ kg–m^2, $\omega = \frac{1000\,(2\,\pi)}{60} = 104.72$ rad/sec,

and

$$\Theta = \frac{500\,(1)}{5000\,(0.25^2 + 0.5^2) - (3.3333 + 50\,(1^2))\,(104.72^2)} = -\,8.5718\,(10^{-4}) \text{ rad}$$

3.25 Derive the equation of motion and find the steady-state solution of the system shown in Fig. 3.45 for rotational motion about the hinge O for the following data: $k = 5000$ N/m, $l = 1$ m, $m = 10$ kg, $M_0 = 100$ N-m, $\omega = 1000$ rpm.



3.25

Equation of motion for rotation about O:

$$J_0\,\ddot{\theta} = -k\,\frac{\theta\,\ell}{4}\,\frac{\ell}{4} - k\,\frac{\theta\,3\,\ell}{4}\,\frac{3\,\ell}{4} + M_0 \cos\omega t$$

$$\text{i.e.,}\quad J_0\,\ddot{\theta} + \left(\frac{5}{8}\,k\,\ell^2\right)\theta = M_0\cos\omega t$$

$$\text{where}\quad J_0 = \frac{1}{12}\,m\,\ell^2 + m\left(\frac{\ell}{4}\right)^2 = \frac{7}{48}\,m\,\ell^2 = \frac{7}{48}\,(10)\,(1^2) = 1.4583\ \text{kg–m}^2$$

and $\omega = 1000$ rpm $= 104.72$ rad/sec. Steady state solution is:

$$\theta_p(t) = \Theta\cos\omega t$$

where

$$\Theta = \frac{M_0}{\frac{5}{8}\,k\,\ell^2 - J_0\,\omega^2} = \frac{100}{5000\left(\frac{5}{8}\right)(1^2) - 1.4583\,(104.72^2)} = -\,0.007772\ \text{rad}$$