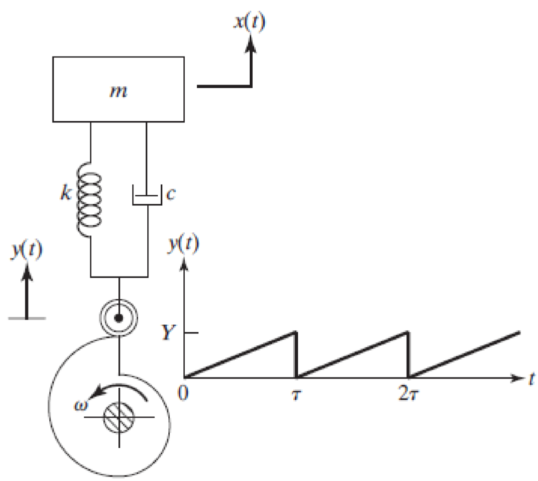
4.8 The base of a spring-mass-damper system is subjected to the periodic displacement shown in Fig. 4.39. Determine the response of the mass using the principle of superposition.



(4.8) Base motion can be represented by Fourier series as (from Example 1.19):

$$y(t) = \frac{Y}{\pi}\left[\frac{\pi}{2} - \left\{\sin \omega t + \frac{1}{2}\sin 2\,\omega t + \frac{1}{3}\sin 3\,\omega t + \cdots\right\}\right]$$

Equation of motion of mass:

$$m\,\ddot{x} + c\,(\dot{x} - \dot{y}) + k\,(x - y) = 0 \qquad (2$$

Since y(t) is composed of several terms, the solution of Eq. (2) can be found b superposingg the solutions corresponding to each of the terms appearing in Eq. (1 When $y(t) = Y/2$, constant, equation of motion becomes:

$$m\,\ddot{x} + c\,\dot{x} + k\,x = \frac{k\,Y}{2} = \text{constant} \qquad (3$$

The steady state solution of Eq. (3) is given by (see Example 4.9):

$$x(t) = \frac{Y}{2}\left[1 - \frac{1}{\sqrt{1-\varsigma^2}}\,e^{-\varsigma\,\omega_n\,t}\cos\,(\omega_d\,t - \phi)\right]$$

$$\text{where}\quad \phi = \tan^{-1}\left(\frac{\varsigma}{\sqrt{1-\varsigma^2}}\right)$$

When $y(t) = A\sin\Omega\,t$, the steady state solution of Eq. (2) is given by Eq. (3.67):

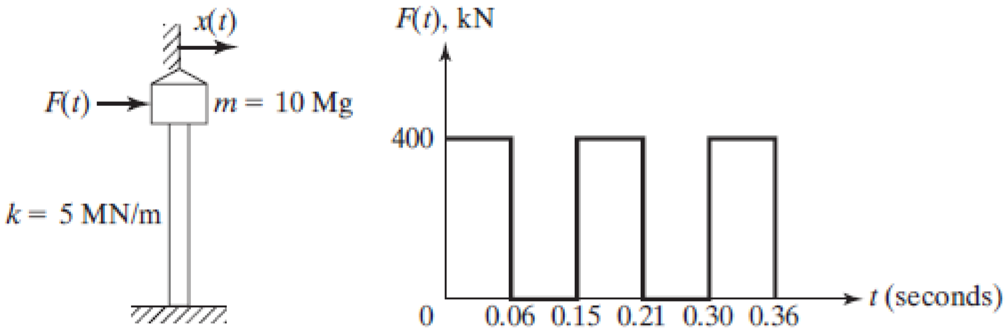
$$x(t) = A\,\sin\,(\Omega\,t - \phi)$$

$$\text{where}\quad A = \left[\frac{1 + (2\,\varsigma\,r)^2}{(1-r^2)^2 + (2\,\varsigma\,r)^2}\right]^{\frac{1}{2}}$$

$$\phi = \tan^{-1}\left(\frac{2\,\varsigma\,r^3}{1 + r^2\,(4\,\varsigma^2 - 1)}\right)$$

$$\text{and}\quad r = \frac{\Omega}{\omega_n}$$

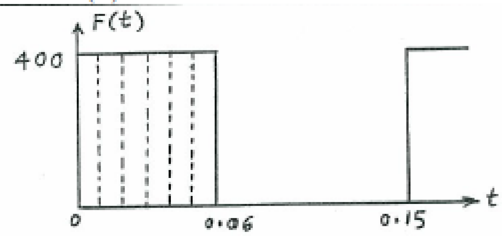
4.15 Find the displacement of the water tank shown in Fig. 4.43(a) under the periodic force shown in Fig. 4.43(b) by treating it as an undamped single-degree-of-freedom system. Use the numerical procedure described in Section 4.3.



(a) (b)

4.15

$\omega_n = \sqrt{\dfrac{5 \times 10^6}{10 \times 10^3}} = 22.3607 \ \dfrac{\text{rad}}{\text{sec}}$



$\tau = 0.15$ sec

$\omega = \dfrac{2\pi}{\tau} = 41.888$ rad/sec

$r = \dfrac{\omega}{\omega_n} = 1.8733$, $r^2 = 3.5093$

$\zeta = 0$

Following data is used in Program 1.F to find Fourier coefficients in the expansion of $F(t)$:

t, sec	0.01	0.02	0.03	0.04	0.05	0.06	0.07	...	0.15
$F(t)$, kN	400	400	400	400	400	400	0	...	0

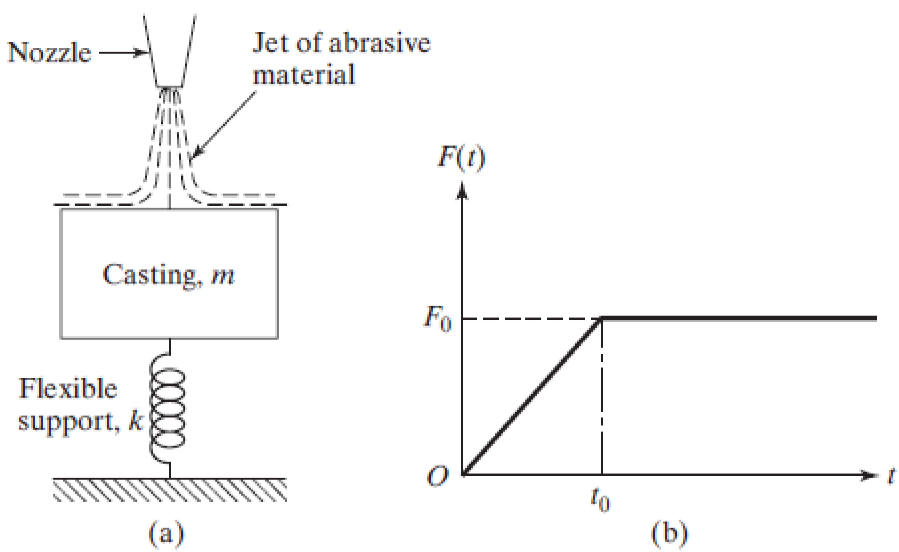
Result is:

$$F(t) = 160.0 + 25.5002 \cos 41.888t + 242.6276 \sin 41.888t$$
$$- 75.3884 \cos 83.776t + 16.0237 \sin 83.776t$$
$$+ 16.4806 \cos 125.664t + 50.7237 \sin 125.664t$$
$$- 62.3538 \cos 167.552t + 27.7604 \sin 167.552t$$
$$+ \cdots \quad \text{kN}$$

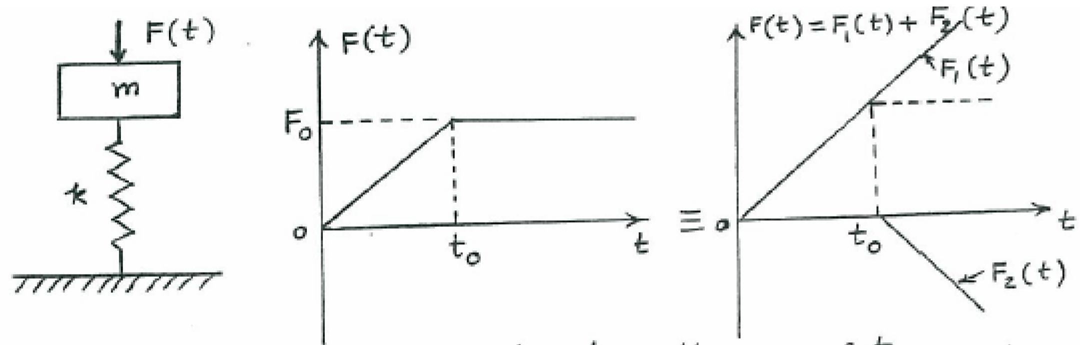
Since $\zeta = 0$ and all $\phi_j = 0$, $j = 1, 2, \cdots$, the steady-state response of the water tank, Eq. (4.13), becomes

$$x_p(t) = 0.032 + 2.0325 \times 10^{-3} \cos 41.888t$$
$$+ 19.3383 \times 10^{-3} \sin 41.888t - 1.1566 \times 10^{-3} \cos 83.776t$$
$$+ 0.2458 \times 10^{-3} \sin 83.776t + 0.1078 \times 10^{-3} \cos 125.664t$$
$$+ 0.3317 \times 10^{-3} \sin 125.664t - 0.2334 \times 10^{-3} \cos 167.552t$$
$$+ 0.1007 \times 10^{-3} \sin 167.552t + \cdots \quad \text{m}$$

4.16 Sandblasting is a process in which an abrasive material, entrained in a jet, is directed onto the surface of a casting to clean its surface. In a particular setup for sandblasting, the casting of mass m is placed on a flexible support of stiffness k as shown in Fig. 4.44(a). If the force exerted on the casting due to the sandblasting operation varies as shown in Fig. 4.44(b), find the response of the casting.



4.16



Forcing function can be considered as the sum of two ramp functions, $F_1(t) = \dfrac{F_0\, t}{t_0}$ and $F_2(t) = -\dfrac{F_0(t-t_0)}{t_0}$.

Response of the casting (undamped spring - mass system) to F_1 is given by

$$x_1(t) = \frac{F_0}{k}\left(\frac{t}{t_0} - \frac{\sin \omega_n t}{\omega_n t_0}\right) \qquad \text{for } t \geq 0 \qquad (E_1)$$

Response due to F_2 can be obtained from Eq. (E_1) by replacing t by $t - t_0$ and F_0 by $-F_0$:

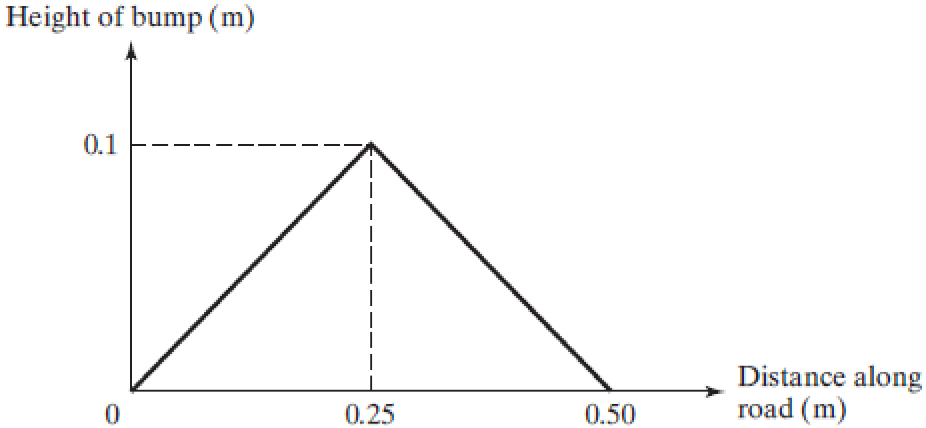
$$x_2(t) = -\frac{F_0}{k}\left\{\frac{t-t_0}{t_0} - \frac{\sin \omega_n(t-t_0)}{\omega_n t_0}\right\} \qquad \text{for } t \geq t_0 \qquad (E_2)$$

Total response of the casting is given by

$$x(t) = x_1(t) + x_2(t) = \frac{F_0}{k}\left\{1 + \frac{\sin \omega_n(t-t_0) - \sin \omega_n t}{\omega_n t_0}\right\}$$

$$\text{for } t \geq t_0 \qquad (E_3)$$

4.25 An automobile, having a mass of 1000 kg, runs over a road bump of the shape shown in Fig. 4.49. The speed of the automobile is 50 km/hr. If the undamped natural period of vibration in the vertical direction is 1.0 sec, find the response of the car by assuming it as a single-degree-of-freedom undamped system vibrating in the vertical direction.

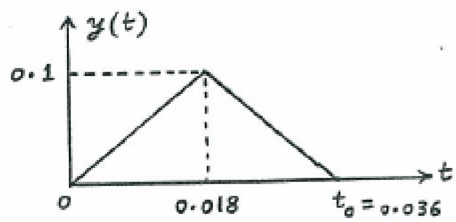


(4.25) Speed of automobile = 50 km/hr

Excitation frequency = $\left(\frac{50 \times 1000}{3600}\right)\frac{1}{0.5} = 27.7778$ Hz

Natural frequency = $f_n = 1.0$ Hz $\Rightarrow$ $\omega_n = 2\pi$ rad/sec

$$t_0 = \frac{0.5 \times 3600}{50 \times 1000} = 0.036 \text{ sec}$$



$$y(t) = \begin{cases} \dfrac{0.2\,t}{t_0} & ; \; 0 \le t \le t_0/2 \\ -\dfrac{0.2\,t}{t_0} + 0.2 & ; \; \dfrac{t_0}{2} \le t \le t_0 \\ 0 & ; \; t > t_0 \end{cases} \quad \cdots (E_1)$$

Equation of motion (for undamped case):

$$m\ddot{x} + k(x-y) = 0 \quad \text{or} \quad m\ddot{x} + kx = ky = F(t) \qquad (E_2)$$

$$\text{where} \quad F(t) = k\,y(t) \qquad (E_3)$$

Solution of Eq. (E_2) is: $$x(t) = \frac{1}{m\omega_n}\int_0^t F(\tau)\,\sin\omega_n(t-\tau)\,d\tau \qquad (E_4)$$

For $0 \le t \le \frac{t_0}{2}$:

$$x(t) = \frac{k}{m\omega_n}\int_0^t \left(\frac{0.2}{t_0}\right)\tau\,\sin\omega_n(t-\tau)\,d\tau \qquad (E_5)$$

Since $\int_0^t \tau\,\sin\omega_n(t-\tau)\,d\tau = \left(\frac{t}{\omega_n} - \frac{1}{\omega_n^2}\sin\omega_n t\right)$,

Eq. (E_5) becomes

$$x(t) = 5.5556\,(t - 0.1592\,\sin 6.2832\,t) \;\; m \;;\; 0 \le t \le 0.018 \text{ sec} \qquad (E_6)$$

For $\frac{t_0}{2} \le t \le t_0$:

$$x(t) = \frac{k}{m\omega_n}\left\{\int_0^{t_0/2}\frac{0.2\tau}{t_0}\,\sin\omega_n(t-\tau)\,d\tau + \int_{t_0/2}^{t}\left(-\frac{0.2\tau}{t_0}+0.2\right)\sin\omega_n(t-\tau)\,d\tau\right\}$$

But

$$\frac{0.2k}{m\omega_n t_0}\cdot\int_0^{t_0/2}\tau\cdot\sin\omega_n(t-\tau)\,d\tau = \frac{0.2k}{m\omega_n t_0}\left\{\sin\omega_n t\left[\frac{1}{\omega_n^2}\cos\omega_n\tau + \frac{\tau}{\omega_n}\sin\omega_n\tau\right]_0^{t_0/2} - \cos\omega_n t\left[\frac{1}{\omega_n^2}\sin\omega_n\tau - \frac{\tau}{\omega_n}\cos\omega_n\tau\right]_0^{t_0/2}\right\}$$

$$= 5.5556\left[0.1592\,\sin 6.2832(t-0.018) + 0.0180\,\cos 6.2832\,(t-0.018) - 0.1592\,\sin 6.2832\,t\right] \; m \qquad (E_7)$$

since $t_0 = 0.036$.

$$-\frac{0.2k}{m\omega_n t_0}\int_{t_0/2}^{t}\tau\,\sin\omega_n(t-\tau)\,d\tau = -\frac{0.2k}{m\omega_n t_0}\left\{\sin\omega_n t\left[\frac{1}{\omega_n^2}\cos\omega_n\tau + \frac{\tau}{\omega_n}\sin\omega_n\tau\right]_{t_0/2}^{t} - \cos\omega_n t\cdot\left[\frac{1}{\omega_n^2}\sin\omega_n\tau - \frac{\tau}{\omega_n}\cos\omega_n\tau\right]_{t_0/2}^{t}\right\}$$

$$= -5.5556\left[t - 0.1592\,\sin 6.2832\,(t-0.018) - 0.0180\,\cos 6.2832\,(t-0.018)\right] \; m \qquad (E_8)$$

$$\frac{0.2\,k}{m\omega_n}\int_{t_0/2}^{t}\sin\omega_n(t-\tau)\,d\tau = \frac{0.2\,k}{m\omega_n}\left[\frac{1}{\omega_n}\cos\omega_n(t-\tau)\right]_{t_0/2}^{t}$$

$$= \frac{0.2\,k}{m\omega_n^2}\left[1-\cos\omega_n\left(t-\frac{t_0}{2}\right)\right]$$

$$= 0.2\left[1-\cos 6.2832\,(t-0.018)\right]\ \text{m} \qquad (E_9)$$

Hence the solution can be expressed as

$$x(t) = \left[1.7689\sin 6.2832\,(t-0.018) - 0.8845\sin 6.2832\,t - 5.5556\,t + 0.2\right]\ \text{m}\ ; \quad 0.018 \le t \le 0.036\ \text{sec} \qquad (E_{10})$$

For $t > t_0$:

$$x(t) = \frac{0.2\,k}{m\omega_n t_0}\int_0^{t_0/2}\tau\sin\omega_n(t-\tau)\,d\tau - \frac{0.2\,k}{m\omega_n t_0}\int_{t_0/2}^{t_0}\tau\sin\omega_n(t-\tau)\,d\tau + \frac{0.2\,k}{m\omega_n}\int_{t_0/2}^{t_0}\sin\omega_n(t-\tau)\,d\tau \qquad (E_{11})$$

The first term on the right side of (E_{11}) is given by (E_7).

Second term on the right side of (E_{11}) is

$$-\frac{0.2\,k}{m\omega_n t_0}\int_{t_0/2}^{t_0}\tau\sin\omega_n(t-\tau)\,d\tau = -\frac{0.2\,k}{m\omega_n t_0}\left\{\sin\omega_n t\cdot\left[\frac{1}{\omega_n^2}\cos\omega_n\tau + \frac{\tau}{\omega_n}\sin\omega_n\tau\right]_{t_0/2}^{t_0} - \cos\omega_n t\cdot\left[\frac{1}{\omega_n^2}\sin\omega_n\tau - \frac{\tau}{\omega_n}\cos\omega_n\tau\right]_{t_0/2}^{t_0}\right\}$$

$$= -5.5556\left[0.1592\sin 6.2832\,(t-0.036) + 0.0360\cos 6.2832\,(t-0.036) - 0.1592\sin 6.2832\,(t-0.018) - 0.0180\cos 6.2832\,(t-0.018)\right]\ \text{m} \qquad (E_{12})$$

The third term on the right side of (E_{11}) is:

$$\frac{0.2\,k}{m\omega_n}\int_{t_0/2}^{t_0}\sin\omega_n(t-\tau)\,d\tau = \frac{0.2\,k}{m\omega_n}\left[\frac{1}{\omega_n}\cos\omega_n(t-\tau)\right]_{t_0/2}^{t_0}$$

$$= \frac{0.2\,k}{m\omega_n^2}\left[\cos\omega_n(t-t_0) - \cos\omega_n\left(t-\frac{t_0}{2}\right)\right]$$

$$= 0.2\left[\cos 6.2832\,(t-0.036) - \cos 6.2832\,(t-0.018)\right]\ \text{m} \qquad (E_{13})$$

$\therefore$ $x(t)$ is given by the sum of Eqs. (E_7), (E_{12}) and (E_{13}), which can be simplified as

$$x(t) = 1.7689 \sin 6.2832 (t - 0.018) - 0.8845 \sin 6.2832\, t$$
$$- 0.8845 \sin 6.2832 (t - 0.036) \quad m \; ; \quad t > 0.036 \text{ sec} \qquad (E_{14})$$