Tu + HW (2 DOF SYSTEMS)

5.1 Find the natural frequencies of the system shown in Fig. 5.16, with $m_1 = m$, $m_2 = 2m$, $k_1 = k$, and $k_2 = 2k$. Determine the response of the system when $k = 1000$ N/m, $m = 20$ kg, and the initial values of the displacements of the masses m_1 and m_2 are 1 and -1, respectively.

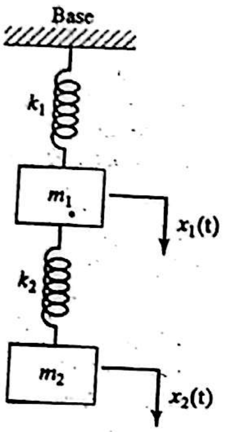

FIGURE 5.16

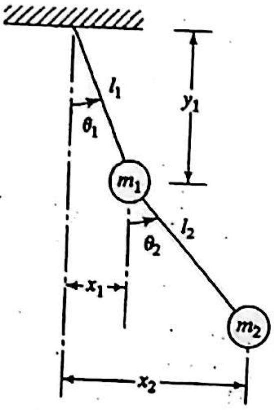

FIGURE 5.17

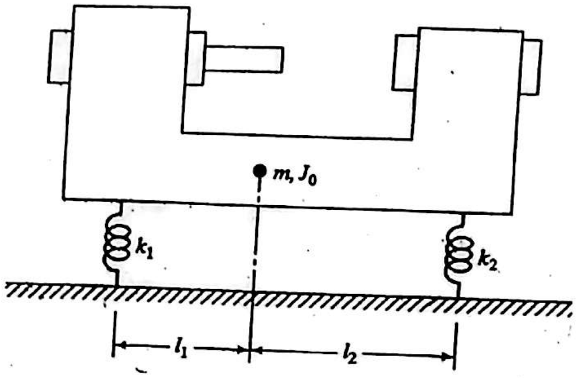

FIGURE 5.19

5.2 Set up the differential equations of motion for the double pendulum shown in Fig. 5.17, using the coordinates x_1 and x_2 and assuming small amplitudes. Find the natural frequencies, the ratios of amplitudes, and the locations of nodes for the two modes of vibration when $m_1 = m_2 = m$ and $l_1 = l_2 = l$.

5.4 A machine tool, having a mass of $m = 1000$ kg and a mass moment of inertia of $J_0 = 300$ kg-m^2, is supported on elastic supports, as shown in Fig. 5.19. If the stiffnesses of the supports are given by $k_1 = 3000$ N/mm, and $k_2 = 2000$ N/mm, and the supports are located at $l_1 = 0.5$ m and $l_2 = 0.8$ m, find the natural frequencies and mode shapes of the machine tool.

5.7 The drilling machine shown in Fig. 5.21(a) can be modeled as a two degree of freedom system as indicated in Fig. 5.21(b). Since a transverse force applied to mass m_1 or mass m_2 causes both the masses to deflect, the system exhibits elastic coupling. The bending stiffnesses of the column are given by (see Section 6.4 for the definition of stiffness influence coefficients)

$$k_{11} = \frac{768}{7}\frac{EI}{l^3}, \qquad k_{12} = k_{21} = -\frac{240}{7}\frac{EI}{l^3}, \qquad k_{22} = \frac{96}{7}\frac{EI}{l^3}$$

Determine the natural frequencies of the drilling machine.

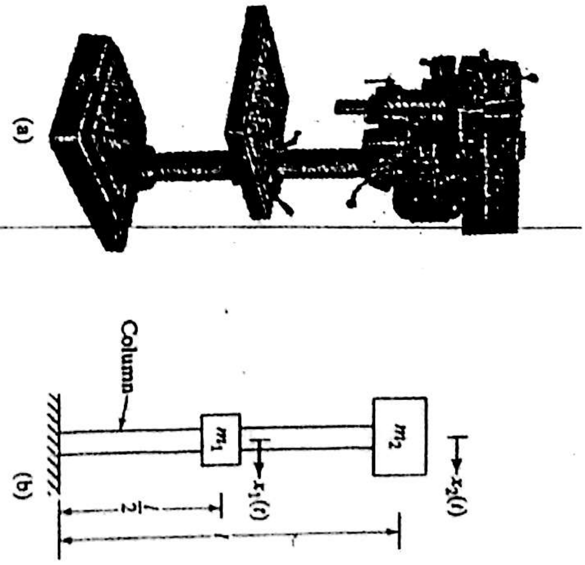

FIGURE 5.21 (Photo courtesy of Atlas-Clausing division.)

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5.21 Determine the initial conditions of the system shown in Fig. 5.16 for which the system vibrates only at its lowest natural frequency for the following data: $k_1 = k$, $k_2 = 2k$, $m_1 = m$, $m_2 = 2m$.

5.22 The system shown in Fig. 5.16 is initially disturbed by holding the mass m_1 stationary and giving the mass m_2 a downward displacement of 0.1 m. Discuss the nature of the resulting motion of the system.

5.27 Two identical pendulums, each with mass m and length l, are connected by a spring of stiffness k at a distance d from the fixed end, as shown in Fig. 5.27.

a. Derive the equations of motion of the two masses.
b. Find the natural frequencies and mode shapes of the system.
c. Find the free vibration response of the system for the initial conditions $\theta_1(0) = a$, $\theta_2(0) = 0$, $\dot{\theta}_1(0) = 0$, and $\dot{\theta}_2(0) = 0$.
d. Determine the condition(s) under which the system exhibits a beating phenomenon.

5.28 Determine the natural frequencies and normal modes of the torsional system shown in Fig. 5.28 for $k_{t2} = 2k_{t1}$ and $J_2 = 2J_1$.

5.29 Determine the natural frequencies of the system shown in Fig. 5.29 by assuming that the rope passing over the cylinder does not slip.

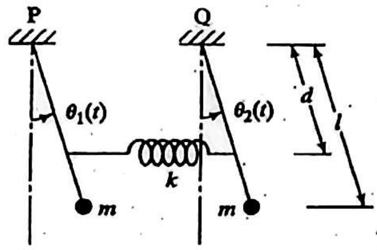

FIGURE 5.27

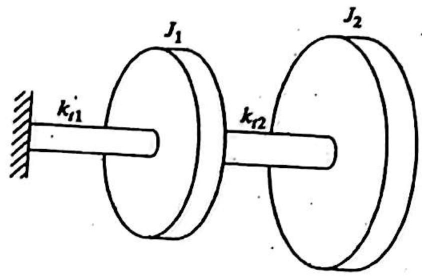

FIGURE 5.28

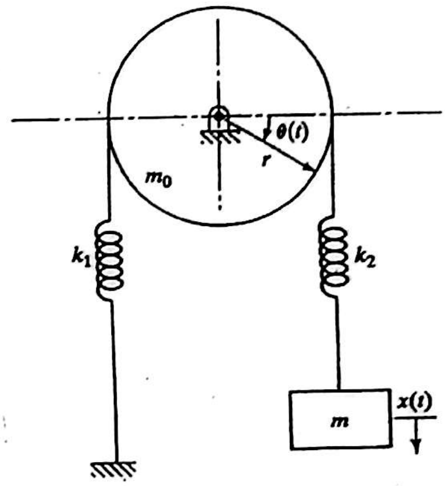

FIGURE 5.29

5.32 A simplified ride model of the military vehicle in Fig. 5.30(a) is shown in Fig. 5.30(b). This model can be used to obtain information about the bounce and pitch modes of the vehicle. If the total mass of the vehicle is m and the mass moment of inertia about its C.G. is J_0, derive the equations of motion of the vehicle using two different sets of coordinates, as indicated in Section 5.5.

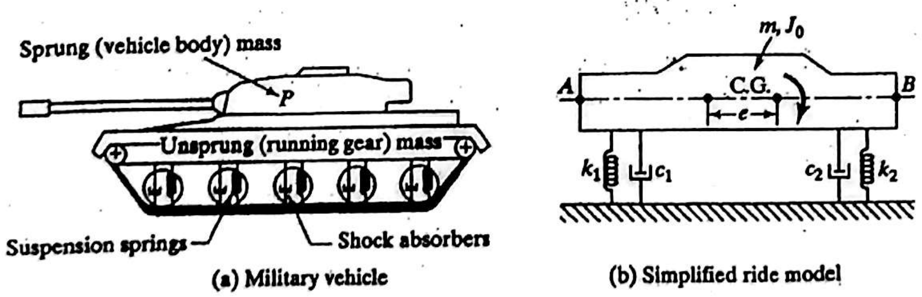

FIGURE 5.30

5.43 A trailer of mass M is connected to a wall through a spring of stiffness k_1 and can move on a frictionless horizontal surface, as shown in Fig. 5.40. A uniform cylinder of mass m, connected to the wall of the trailer by a spring of stiffness k_2, can roll on the floor of the trailer without slipping. Derive the equations of motion of the system and discuss the nature of coupling present in the system.

FIGURE 5.40