

Operations Research

Practice Problems: Formulation and Graphical Solution Procedure

- 1) The World Light Company produces two light fixtures (products 1 and 2) that require both metal frame parts and electrical components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of \$1, and each unit of product 2, up to 60 units, gives a profit of \$2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.
 - (a) Formulate a linear programming model for this problem.
 - (b) Use the graphical method to solve this model. What is the resulting total profit?

- 2) The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Work-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

- (a) Formulate a linear programming model for this problem.
 - (b) Use the graphical method to solve this model.

- 3) Weenies and Buns is a food processing plant which manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires $\frac{1}{4}$ pound of pork product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each hot dog bun requires 2 minutes of labor. Each hot dog yields a profit of \$0.20, and each bun yields a profit of \$0.10. Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit.
 - (a) Formulate a linear programming model for this problem.
 - (b) Use the graphical method to solve this model.

- 4) The Omega Manufacturing Company has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

Machine Type	Available Time (Machine Hours per Week)
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is

Productivity coefficient (in machine hours per unit)

Machine Type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be \$50, \$20, and \$25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product Omega should produce to maximize profit. Formulate a linear programming model for this problem.

- 5) This is your lucky day. You have just won a \$10,000 prize. You are setting aside \$4,000 for taxes and partying expenses, but you have decided to invest the other \$6,000. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, and your estimated profit (ignoring the value of your time) would be \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any *fraction* of a full partnership you would like. If you choose a fraction of a full partnership, all the above figures given for a full partnership (money investment, time investment, and your profit) would be multiplied by this same fraction. Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.
- (a) Formulate a linear programming model for this problem.
- (b) Use the graphical method to solve this model. What is your total estimated profit?
- 6) Dwight is an elementary school teacher who also raises pigs for supplemental income. He is trying to decide what to feed his pigs. He is considering using a combination of pig feeds available from local suppliers. He would like to feed the pigs at minimum cost while also making sure each pig receives an adequate supply of calories and vitamins. The cost, calorie content, and vitamin content of each feed is given in the table below.

Contents	Feed Type A	Feed Type B
Calories (per pound)	800	1,000
Vitamins (per pound)	140 units	70 units
Cost (per pound)	\$0.40	\$0.80

Each pig requires at least 8,000 calories per day and at least 700 units of vitamins. A further constraint is that no more than one-third of the diet (by weight) can consist of Feed Type A, since it contains an ingredient which is toxic if consumed in too large a quantity.

- (a) Formulate a linear programming model for this problem.
- (b) Use the graphical method to solve this model. What is the resulting daily cost per pig?
- 7) Web Mercantile sells many household products through an on-line catalogue. The company needs substantial warehouse space for storing its goods. Plans now are being made for leasing warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease

expire) at least once but not every month. The space requirement and the leasing costs for the various leasing periods are as follows:

Month	Required Space (Sq. Ft.)	Leasing Period (Months)	Cost per Sq. Ft. Leased
1	30,000	1	\$ 65
2	20,000	2	\$100
3	40,000	3	\$135
4	10,000	4	\$160
5	50,000	5	\$190

The objective is to minimize the total leasing cost for meeting the space requirements. Formulate a linear programming model for this problem.

- 8) Larry Edison is the director of the Computer Center for Buckley College. He now needs to schedule the staffing of the center. It is open from 8 A.M. until midnight. Larry has monitored the usage of the center at various times of the day, and determined that the following number of computer consultants is required:

Time of Day	Minimum Number of Consultants Required to Be on Duty
8 A.M.–noon	4
Noon–4 P.M.	8
4 P.M.–8 P.M.	10
8 P.M.–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for 8 consecutive hours in any of the following shifts: morning (8 A.M.–4 P.M.), afternoon (noon–8 P.M.), and evening (4 P.M.–midnight). Full-time consultants are paid \$14 per hour. Part-time consultants can be hired to work any of the four shifts listed in the above table. Part-time consultants are paid \$12 per hour. An additional requirement is that during every time period, there must be at least 2 full-time consultants on duty for every part-time consultant on duty. Larry would like to determine how many full-time and how many part-time workers should work each shift to meet the above requirements at the minimum possible cost. Formulate a linear programming model for this problem.

- 9) Oxbridge University maintains a powerful mainframe computer for research use by its faculty, Ph.D. students, and research associates. During all working hours, an operator must be available to operate and maintain the computer, as well as to perform some programming services. Beryl Ingram, the director of the computer facility, oversees the operation. It is now the beginning of the fall semester, and Beryl is confronted with the problem of assigning different working hours to her operators. Because all the operators are currently enrolled in the university, they are available to work only a limited number of hours each day, as shown in the following table.

Operators	Wage Rate	Maximum Hours of Availability				
		Mon.	Tue.	Wed.	Thurs.	Fri.
K. C.	\$10.00/hour	6	0	6	0	6
D. H.	\$10.10/hour	0	6	0	6	0
H. B.	\$ 9.90/hour	4	8	4	0	4
S. C.	\$ 9.80/hour	5	5	5	0	5
K. S.	\$10.80/hour	3	0	3	8	0
N. K.	\$11.30/hour	0	0	0	6	2

There are six operators (four undergraduate students and two graduate students). They all have different wage rates because of differences in their experience with computers and in their programming ability. The above table shows their wage rates, along with the maximum number of hours that each can work each day. Each operator is guaranteed a certain minimum number of hours per week that will maintain an

adequate knowledge of the operation. This level is set arbitrarily at 8 hours per week for the undergraduate students (K. C., D. H., H. B., and S. C.) and 7 hours per week for the graduate students (K. S. and N. K.). The computer facility is to be open for operation from 8 A.M. to 10 P.M. Monday through Friday with exactly one operator on duty during these hours. On Saturdays and Sundays, the computer is to be operated by other staff. Because of a tight budget, Beryl has to minimize cost. She wishes to determine the number of hours she should assign to each operator on each day. Formulate a linear programming model for this problem.

- 10) Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches. They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content of each food choice and its cost are given in the table below.

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (€)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp)	75	100	0	4	4
Strawberry jelly (1 tbsp)	0	50	3	0	7
Graham cracker (1 cracker)	20	60	0	1	8
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice). Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements. Formulate a linear programming model for this problem.

- 11) Maureen Laird is the chief financial officer for the Alva Electric Co., a major public utility in the midwest. The company has scheduled the construction of new hydroelectric plants 5, 10, and 20 years from now to meet the needs of the growing population in the region served by the company. To cover at least the construction costs, Maureen needs to invest some of the company's money now to meet these future cash-flow needs. Maureen may purchase only three kinds of financial assets, each of which costs \$1 million per unit. Fractional units may be purchased. The assets produce income 5, 10, and 20 years from now, and that income is needed to cover at least minimum cash-flow requirements in those years. (Any excess income above the minimum requirement for each time period will be used to increase dividend payments to shareholders rather than saving it to help meet the minimum cash-flow requirement in the next time period.) The following table shows both the amount of income generated by each unit of each asset and the minimum amount of income needed for each of the future time periods when a new hydroelectric plant will be constructed.

Year	Income per Unit of Asset			Minimum Cash Flow Required
	Asset 1	Asset 2	Asset 3	
5	\$2 million	\$1 million	\$0.5 million	\$400 million
10	\$0.5 million	\$0.5 million	\$1 million	\$100 million
20	0	\$1.5 million	\$2 million	\$300 million

Maureen wishes to determine the mix of investments in these assets that will cover the cash-flow requirements while minimizing the total amount invested. Formulate a linear programming model for this problem.