Spring 2017-18

Operations Research **Class Test 1** **Marks: 10**

(1) Automobile traffic from three highways, H1, H2, and H3, must stop and wait for a green light before exiting to a toll road. The tolls are \$3, \$4, and \$5 for cars exiting from H1, H2, and H3, respectively. The traffic flow rates from H1, H2, and H3 are 500, 600, and 400 cars per hour. The traffic light cycle should not exceed 132 seconds, and the green light on any highway must be at least 25 seconds. The yellow light is on for 10 seconds. The toll gate can handle a maximum of 510 cars per hour. Assuming no cars move on yellow, formulate a linear programming problem to find the optimal green time interval for the three highways that will maximize toll gate revenue per traffic cycle. [4]

Let t_i → green time interval in sec. for highway i, $i=1,2,3$

$$\max z = 3\left(\frac{500}{3600}\right)t_1 + 4\left(\frac{600}{3600}\right)t_2 + 5\left(\frac{400}{3600}\right)t_3$$

s.t. $t_1 + t_2 + t_3 + 3\times 10 \le 132$: (traffic light cycle)

$t_1 \geq 25,\ t_2 \geq 25,\ t_3 \geq 25$: (green light)

$$\left(\frac{500}{3600}\right)t_1 + \left(\frac{600}{3600}\right)t_2 + \left(\frac{400}{3600}\right)t_3 \le \frac{510}{3600}(t_1+t_2+t_3)$$

: (max capacity)

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(2) Maximize $Z = x_1 + 3x_2$ [3]

Subject to

$x_1 + x_2 \le 2$ (resource 1) $x_1 + x_2 + x_3 = 2$

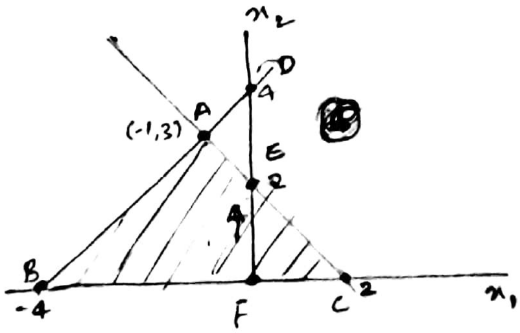
$-x_1 + x_2 \le 4$ (resource 2) $-x_1 + x_2 + x_4 = 4$

x_1 unrestricted and $x_2 \ge 0$

(a) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(b) Solve the problem graphically to find the optimal solution of the problem.

	Combination (Basic var.)	Sol^n for Basic var	Status	z
A	x_1, x_2	-1, 3	Feasible	8
B	x_1, x_3	-4, 6	Feasible	-4
C	x_1, x_4	2, 6	Feasible	2
D	x_2, x_3	4, -2	Infeasible	—
E	x_2, x_4	2, 2	Feasible	6
F	x_3, x_4	2, 4	Feasible	0



optimal sol^n $(-1,3)$, $z = 8$

(3) Consider the following LP [3]

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(a) If the problem is solved using Big-M method, write the initial simplex table, and identify the entering and leaving variables.

(b) If x_3 and x_4 are the initial basic variables, apply simplex algorithm to find the optimal solution of the problem.

(i) $\max z' = 2x_1 + 4x_2 + 4x_3 - 3x_4 - M(\bar{x}_5 + \bar{x}_6)$

s.t. $x_1 + x_2 + x_3 + \bar{x}_5 = 4$

$x_1 + 4x_2 + x_4 + \bar{x}_6 = 8$

1/2

BV	x_1	x_2 ↓	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS	Ratio
$\bar{x}_5$	1	1	1	0	1	0	4	4
$\bar{x}_6$	1	4	0	1	0	1	8	2 →
z'	-2	-4	-4	3	M	M	0	← Not in proper form
z'	-(2+2M)	-(4+5M) ↓	-(4+M)	3-M	0	0	-12M	

Entering → x_2.

Leaving → $\bar{x}_6$

(ii)

1/2

BV	x_1	x_2 ↓	x_3	x_4	RHS	Ratio
x_3	1	1	1	0	4	4
x_4	1	4	0	1	8	2 →
z	-2	-4	-4	3	0	← Not in proper form
z	-1	-12	0	0	-8	
x_3	3/4	0	1	-1/4	2	
x_2	1/4	1	0	1/4	2	
z	2	0	0	3	16	

optimal solⁿ: $x_2 = 2,\ x_3 = 2$

$x_1 = x_4 = 0$

$z = 16.$