Department of Industrial & Systems Engineering
Indian Institute of Technology, Kharagpur
Mid-Semester Examination – Spring 2018
Operations Research (IM41082)
(Attempt ALL SIX questions)

Time: 2 Hrs **Full Mark: 30**

Question 1 [4]

Each day, workers at a Toll Plaza work two 6-hour shifts chosen from 12 A.M. to 6 A.M., 6 A.M. to 12 P.M., 12 P.M. to 6 P.M., and 6 P.M. to 12 A.M. The following number of workers is needed during each shift: 12 A.M. to 6 A.M. — 15 workers; 6 A.M. to 12 P.M.—5 workers; 12 P.M. to 6 P.M.—12 workers; 6 P.M. to 12 A.M. — 6 workers. Workers whose two shifts are consecutive are paid $12 per hour; workers whose shifts are not consecutive are paid $18 per hour. Formulate an LP that can be used to minimize the cost of meeting the daily workforce demands of the Toll Plaza.

Question 2 [1+1+2+1]

Consider the following LP model

Maximize $Z = 5x_1 + 4x_2$

Subject to

$$6x_1 + 4x_2 \le 24$$
$$6x_1 + 3x_2 \le 22.5$$
$$x_1 + x_2 \le 5$$
$$x_1 + 2x_2 \le 6$$
$$-x_1 + x_2 \le 1$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0$$

(a) Use graphical method to find the optimal solution of the problem

(b) Identify the redundant constraint(s), if any.

(c) Find the shadow price of constraints: $x_1 + 2x_2 \le 6$ and $-x_1 + x_2 \le 1$. Also, find the range of RHS (b_4 and b_5) of each of these two resources, in which their shadow prices remain valid.

(d) Find the range of c_1 (profit coefficient of x_1) for which the current solution remains optimal.

Question 3 [1+2+1+1+1]

Consider the following LPP

Maximize $Z = 2x_1 - x_2 + x_3 + x_4$

Subject to

$$-x_1 + x_2 + x_3 + x_5 = 1$$
$$x_1 + x_2 + x_4 = 2$$
$$2x_1 + x_2 + x_3 + x_6 = 6$$
$$x_1, x_2, \ldots, x_6 \ge 0$$

(a) Write down the initial basic feasible solution by inspection.

(b) Find a feasible solution by increasing the nonbasic variable x_1 by one unit, while holding x_2 and x_3 as zero. What will be net change in the objective function?

(c) What is the maximum increase in x_1 possible, subject to the constraints?

(d) Find the new basic feasible solution when x_1 is increased to its maximum value found in (c).

(e) Is the new basic feasible solution obtained in (d) optimal? Why or why not?

Question 4 [4]

Solve the following problem by the Big-M simplex method.

Maximize $3x_1 + x_2$

Subject to: $x_1 - x_2 \le -1$

$-x_1 - x_2 \le -3$

$2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0$

Question 5 [2+1+2]

Consider the following linear program

Minimize $3x_1 + 6x_2 + 2x_3$

s.t. $x_1 + 3x_2 + 2x_3 \ge 6$

$2x_1 + x_2 + x_3 \ge 3$

$x_1, x_2, x_3 \ge 0$

(a) Construct and solve the phase I problem.

(b) Identify the basic variables for the starting of Phase II

(c) Find the optimal solution of phase II

Question 6 [5+1]

Consider the following linear program

Maximize $10x_1 + 12x_2 + 12x_3$

s.t. $x_1 + 2x_2 + 2x_3 \le 20$

$2x_1 + x_2 + 2x_3 \le 20$

$2x_1 + 2x_2 + x_3 \le 20$

$x_1, x_2, x_3 \ge 0$

(a) Solve this LP using revised simplex or in matrix form of simplex

(b) State the nature of the solution i.e. unique, infeasible, unbounded or multiple and justify it.

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