(1)

Let Shift 1 = 12AM-6AM, Shift 2 = 6AM-12PM, Shift 3 = 12PM-6PM, Shift 4 = 6PM-12AM.
Let xij = workers working shifts *i* and *j*

Min $Z = 144(x_{12} + x_{14} + x_{23} + x_{34}) + 216(x_{13} + x_{24})$

Subject to

$x_{12} + x_{13} + x_{14} \geq 15$

$x_{12} + x_{23} + x_{24} \geq 5$

$x_{13} + x_{23} + x_{34} \geq 12$

$x_{14} + x_{24} + x_{34} \geq 6$

All variables ≥ 0

(a) (3, 1·5), Z = 21

(b) Redundant constraints: (1) and (3)

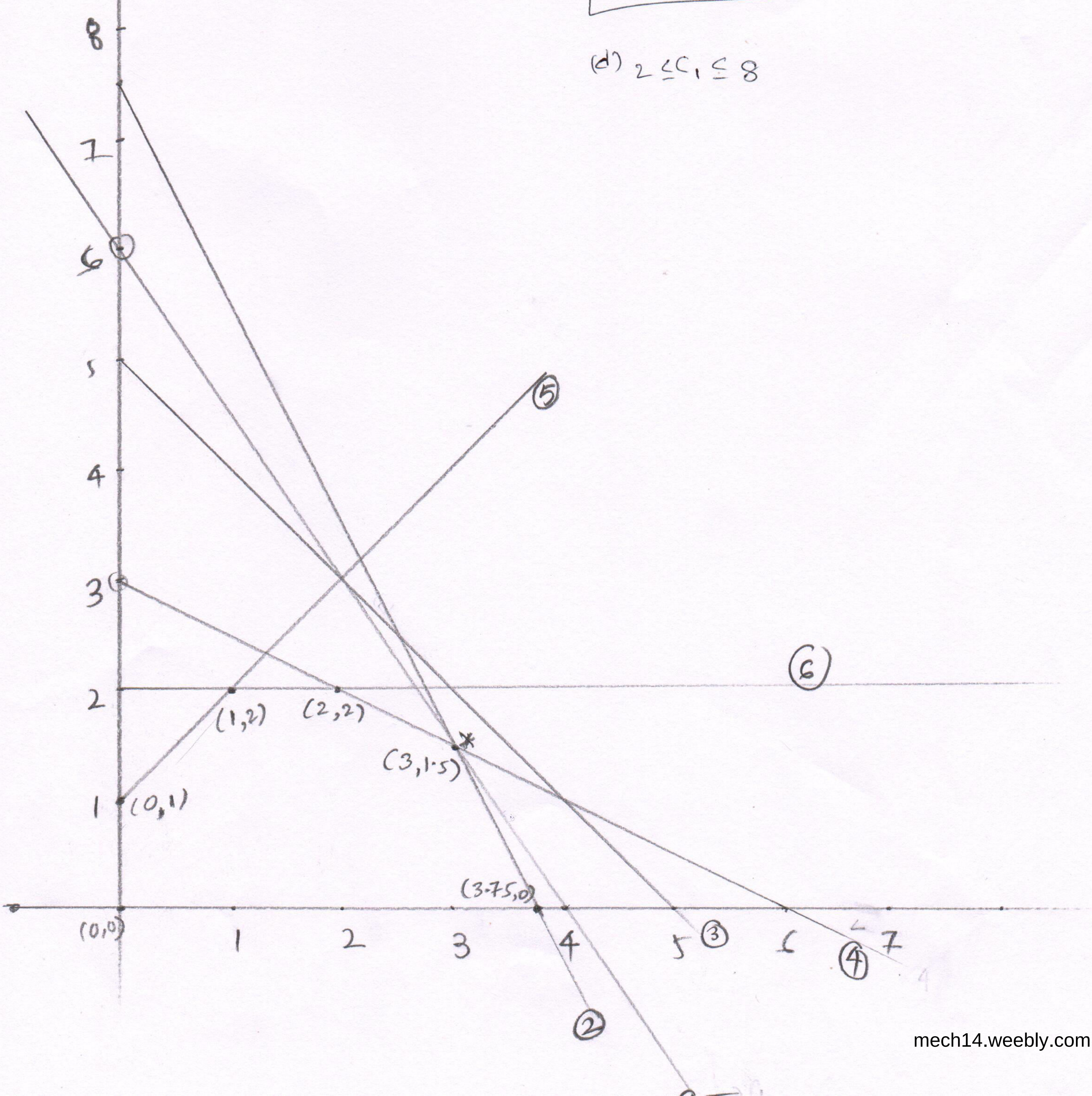
(c) shadow price for resource 4 = 1·0

" " 5 = 0

$\boxed{3.75 \le b_4 \le 6}$

$\boxed{-1.5 \le b_5 \le \infty}$

(d) $2 \le c_1 \le 8$



(3)

a) $x_4=2$, $x_5=1$, $x_6=6$, $x_1=x_2=x_3=0$.

b) $x_1=1$, $x_4=1$, $x_5=2$, $x_6=4$, $x_2=x_3=0$ is the basic feasible solution. Net change in Z = 3-2 = 1.

c) Max $x_1 = Min\ (\infty, \frac{2}{1}, \frac{6}{2}) = 2$.

d) $x_1=2$, $x_5=3$, $x_6=2$, $x_2=x_3=x_4=0$ is the new basic feasible solution.

e) No. It is not optimal, because the relative profit for the nonbasic variable x_3 is 1. Hence Z can be increased further.

(4)

Modified problem

$$\text{Max } z = 3x_1 + x_2 - M\bar{x}_6 - M\bar{x}_7$$

s.t.

$$-x_1 + x_2 - x_3 + \bar{x}_6 = 1$$

$$x_1 + x_2 - x_4 + \bar{x}_7 = 3$$

$$2x_1 + x_2 + x_5 = 4$$

$$x\text{'s} \geq 0,\ \bar{x}_6, \bar{x}_7 \geq 0$$

BV	x_1	x_2 ↓	x_3	x_4	x_5	$\bar{x}_6$	$\bar{x}_7$	RHS	Ratio
$\bar{x}_6$	-1	1	-1	0	0	1	0	1	1 →
$\bar{x}_7$	1	1	0	-1	0	0	1	3	3
x_5	2	1	0	0	1	0	0	4	4
Z	-3	-1	0	0	0	M	M	0	not in proper form
Z	-3 ↓	-1-2M	M	M	0	0	0	-4M	$R_0 \to R_0 - MR_1 - MR_2$
x_2	-1	1	-1	0	0	1	0	1	—
$\bar{x}_7$	2	0	1	-1	0	-1	1	2	1 →
x_5	3	0	1	0	1	-1	0	3	1
Z	-(4+2M)	0	-(1+M)	M ↓	0	(1+2M)	0	1-2M	
x_2	0	1	-1/2	-1/2	0	1/2	1/2	2	—
x_1	1	0	1/2	-1/2	0	-1/2	1/2	1	—
x_5	0	0	-1/2	3/2	1	1/2	-3/2	0	0 →
Z	0	0	1	-2	0	M-1	M+2	5	
x_2	0	1	-2/3	0	1/3	2/3	0	2	
x_1	1	0	1/3	0	1/3	-1/3	0	1	
x_4	0	0	-1/3	1	2/3	1/3	-1	0	
Z	0	0	1/3	0	4/3	(M-1/3)	M	5	

$X_1 = 1$

$z_{max.} = 5$

Q.5) Sol^n $\text{Min } z = 3x_1 + 6x_2 + 2x_3$

S.t :-

$$x_1 + 3x_2 + 2x_3 \geq 6$$

$$2x_1 + x_2 + x_3 \geq 3$$

$$x_1, x_2, x_3 \geq 0.$$

Modified problem :-

$$\text{Min. } z = 3x_1 + 6x_2 + 2x_3 + \cancel{0.x_4 + 0.x_5 + M.\bar{x}_6 + M.\bar{x}_7}.$$

S.t :-

$$x_1 + 3x_2 + 2x_3 - x_4 + \bar{x}_6 = 6$$

$$2x_1 + x_2 + x_3 - x_5 + \bar{x}_7 = 3$$

$$x_1, x_2, x_3, x_4, x_5, \bar{x}_6, \bar{x}_7 \geq 0.$$

Phase I :-

$$\text{Min } z' = \bar{x}_6 + \bar{x}_7$$

$$\text{Min } (-z)' = -\bar{x}_6 - \bar{x}_7.$$

1

S.t :-

$$x_1 + 3x_2 + 2x_3 - x_4 + \bar{x}_6 = 6$$

$$2x_1 + x_2 + x_3 - x_5 + \bar{x}_7 = 3$$

$$x_1, x_2, x_3, x_4, x_5, \bar{x}_6, \bar{x}_7 \geq 0.$$

B.V	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	$\bar{x}_7$	RHS	Ratio
$\bar{x}_6$	1	$\boxed{3}$	2	−1	0	1	0	6	2 →
$\bar{x}_7$	2	1	1	0	−1	0	1	3	3
$-z'$	0	0	0	0	0	1	1	0	
$-z'$	−3	−4	−3	1	1	0	0	−9	
x_2	1/3	1	2/3	−1/3	0	1/3	0	2	6
$\bar{x}_7$	$\boxed{5/3}$	0	1/3	1/3	−1	−1/3	1	1	3/5 →
$-z'$	$\boxed{-5/3}$	0	−1/3	−1/3	1	4/3	0	−1	
x_2	0	1	3/5	−2/5	1/5	2/5	−1/5	9/5	
x_1	1	0	1/5	1/5	−3/5	−1/5	3/5	3/5	
$-z'$	0	0	0	0	0	1	1	0	

Min $z' = 0$

$x_1 = 0$, $x_2 = 0$

Basic variables; $x_2 = 9/5$, $x_1 = 3/5$

Phase II :-

$$\text{Min } z = 3x_1 + 6x_2 + 2x_3.$$

mech14.weebly.com

B.V	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
x_2	0	1	$\boxed{3/5}$	$-2/5$	$1/5$	$9/5$	3 →
x_1	1	0	$1/5$	$1/5$	$-3/5$	$3/5$	3
z'	3	6	2	0	0	0	
z'	0	0	$-11/5$	$9/5$	$3/5$	$-63/5$	
x_3	0	$5/3$	1	$-2/3$	$1/3$	3	
x_1	1	$-1/3$	0	$1/3$	$-2/3$	0	
z'	0	$11/3$	0	$1/3$	$4/3$	-6	

(2)

$$\boxed{x_1 = 0,\ x_2 = 0,\ x_3 = 3,\ Z_{min} = 6}$$

Ans.

if, we leave x_1 :—

B.V	x_1	x_2	x_3	x_4	x_5	RHS	Ratio
x_2	-3	1	0	-1	$\boxed{2}$	0	0 →
x_3	5	0	1	1	-3	3	-1
z'	11	0	0	4	-6	-6	
x_5	$-3/2$	$1/2$	0	$-1/2$	1	0	
x_3	$1/2$	$3/2$	1	$-1/2$	0	3	
z'	2	3	0	1	0	-6	

$Z_{min} = 6$

Q. 6

Augmented form

$\max z = 10x_1 + 12x_2 + 12x_3$

s.t. $x_1 + 2x_2 + 2x_3 + x_4 = 20$

$2x_1 + x_2 + 2x_3 + x_5 = 20$

$2x_1 + 2x_2 + x_3 + x_6 = 20$

$x's \geq 0$

$$\underset{\sim}{c} = [10 \quad 12 \quad 12]$$

$$[\underset{\sim}{A} \; \underset{\sim}{I}] = \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \qquad \underset{\sim}{b} = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

Initial

$$\underset{\sim}{x_B} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad \underset{\sim}{B} = B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \underset{\sim}{B^{-1}} \underset{\sim}{b} = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}, \quad \underset{\sim}{c_B} = [0 \quad 0 \quad 0]$$

$$\underset{\sim}{c_B} \underset{\sim}{B^{-1}} \underset{\sim}{A} - \underset{\sim}{c} = [-10 \quad -12 \quad -12], \quad \underset{\sim}{c_B} \underset{\sim}{B^{-1}} = [0 \quad 0 \quad 0]$$

Tie in entering variable

$\Rightarrow x_2$ entering variable

$$\text{Ratio} = \begin{bmatrix} 20/2 \\ 20/1 \\ 20/2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 10 \end{bmatrix} \rightarrow \quad \text{tie for leaving variable}$$

Let x_4 leaving variable

Iteration 1

$$\underset{\sim}{x_B} = \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix}, \quad \underset{\sim}{B} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow \underset{\sim}{B^{-1}} = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$B^{-1}a_2 =$

$$\underset{\sim}{c_B} = [12 \quad 0 \quad 0]$$

$$x_B = \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix} = \underset{\sim}{B^{-1}} \underset{\sim}{b} = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{c_B} \underset{\sim}{B^{-1}} \underset{\sim}{A} - \underset{\sim}{c} = [12 \quad 0 \quad 0] \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - [10 \quad 12 \quad 12]$$

$\text{Ratio} = \cancel{[\,10}$ $\qquad \underset{\sim}{B}^{-1}\underset{\sim}{a}_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1/2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ 1 \end{bmatrix}$

$$\text{Ratio} = \begin{bmatrix} 10/1/2 \\ 10/3/2 \\ 0/1 \end{bmatrix} = \begin{bmatrix} 20 \\ 20/3 \\ 0 \end{bmatrix} \Rightarrow x_6 \text{ Leaving var}$$

Iteration 2

$$\underset{\sim}{x}_B = \begin{bmatrix} x_2 \\ x_5 \\ x_1 \end{bmatrix}, \quad \underset{\sim}{B} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}, \underset{\sim}{B}^{-1} = \begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$c_B = [12 \quad 0 \quad 10]$$

$$\underset{\sim}{x}_B = \begin{bmatrix} x_2 \\ x_5 \\ x_1 \end{bmatrix} = \underset{\sim}{B}^{-1}\underset{\sim}{b} = \begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{c}_B\underset{\sim}{B}^{-1}\underset{\sim}{A} - \underset{\sim}{c} = [12 \quad 0 \quad 10]\begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - [10 \quad 12 \quad 12]$$

$$= [0 \quad 0 \quad -4] \Rightarrow x_3 \text{ entering variable}$$

$$\underset{\sim}{c}_B\underset{\sim}{B}^{-1} = [2 \quad 0 \quad 4]$$

$$B^{-1}a_3 = \begin{bmatrix} 1 & 0 & -1/2 \\ 1 & 1 & -3/2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 5/2 \\ -1 \end{bmatrix}$$

$$\text{Ratio} = \begin{bmatrix} 10/3/2 \\ 10/5/2 \\ - \end{bmatrix} = \begin{bmatrix} 20/3 \\ 20/5 \\ - \end{bmatrix} \Rightarrow x_5 \text{ Leaving variable}$$

Iteration 3

$$\underset{\sim}{x}_B = \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}, \quad \underset{\sim}{B} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix}, \underset{\sim}{B}^{-1} = \begin{bmatrix} 2/5 & -3/5 & 2/5 \\ 2/5 & 2/5 & -3/5 \\ -3/5 & 2/5 & 2/5 \end{bmatrix}$$

$$\underset{\sim}{c}_B = [12 \quad 12 \quad 10], \quad \underset{\sim}{x}_B = \underset{\sim}{B}^{-1}\underset{\sim}{b} = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}, \quad z = \underset{\sim}{c}_B\underset{\sim}{B}^{-1}\underset{\sim}{b} = 136$$