Shadow Price

Objective Max Type and functional constraint ≤ type

Shadow price for resource i: the rate at which Z could be increased by (slightly) increasing the amount b_i of resource i. The increase in b_i must be sufficiently small that the current Basis (set of basic variables) remains optimal.

- (In the case of a functional constraint in ≥ or = form, its shadow price is again defined as the rate at which Z could be increased by (slightly) increasing the value of b_i, although the interpretation of bi now would normally be something other than the amount of a resource being made available.)
- $b_i \rightarrow$ available amount of resource i

  $b_i \rightarrow b_i + \Delta b_i$ (Suppose b_i can be increased by a small amount)

- **Graphical approach to find Shadow Price**

  Tech Edge Co. Problem:

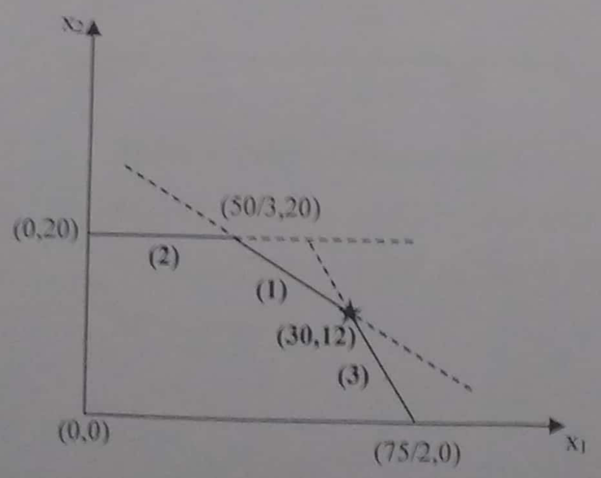
Max $Z = 50x_1+40x_2$

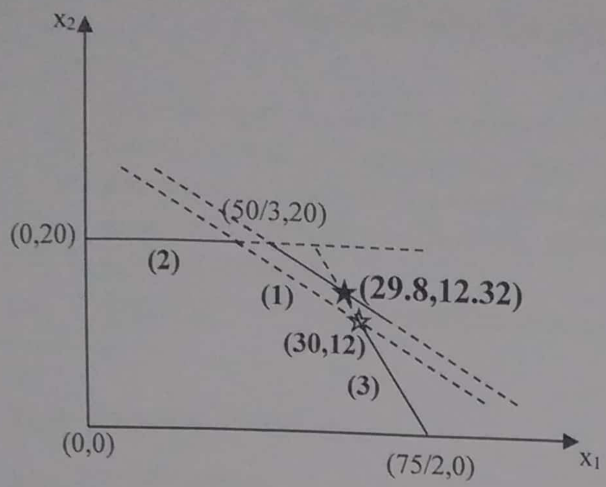
S.t. $3x_1+5x_2 \le 150$ (Assembly time) **(1)**

$x_2 \le 20$ (Special Display unit) **(2)**

$8x_1+5x_2 \le 300$ (Warehouse Space) **(3)**

$x_1, x_2 \ge 0$



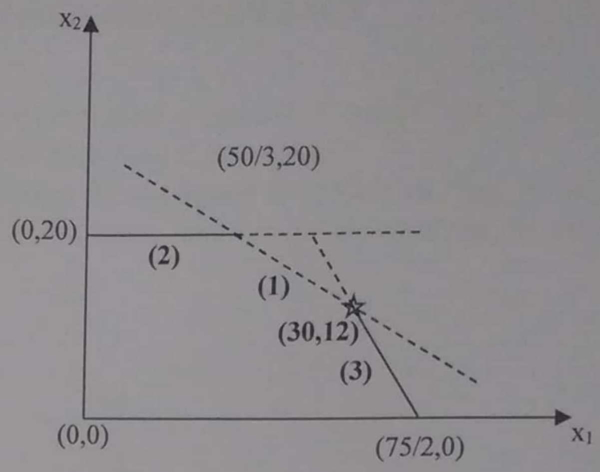


Shadow Price of Resource (1)

$3x_1+5x_2 \leq 150 \rightarrow$ (30,12), Z=1980

$3x_1+5x_2 \leq 151 \rightarrow$ (29.8,12.32), Z=1982.8

$\Delta Z=2.8=y_1^*$

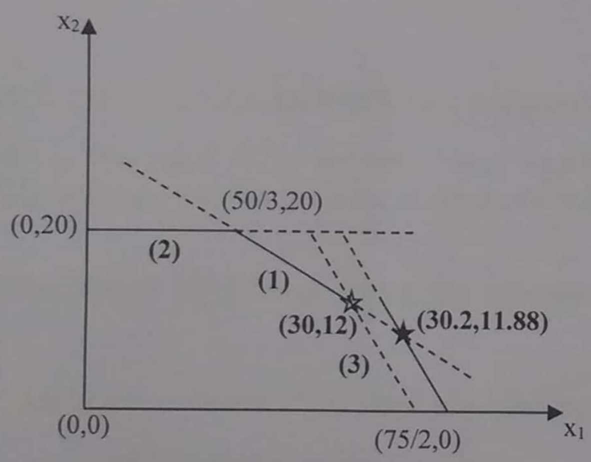


Shadow Price of Resource (2)

$x_2 \leq 20 \rightarrow$ (30,12), Z=1980

$x_2 \leq 21 \rightarrow$ (30,12), Z=1980

$\Delta Z=0=y_2^*$



Shadow Price of Resource (3)

$8x_1+5x_2 \leq 300 \rightarrow$ (30,12), Z=1980

$8x_1+5x_2 \leq 301 \rightarrow$ (30.2,11.88), Z=1985.2

$\Delta Z=5.2=y_3^*$

Sensitivity Analysis: Graphical approach

- How changes in an LP's parameters affect the optimal solution? => sensitivity analysis
- Consider violation of certainty assumptions about model parameters
- **Purpose:** to identify the **sensitive parameters by studying the** effect of changes in the parameters (profit coefficients, resource availability, and resource requirement) on the optimal solution.
  i.e., if $y_i^* > 0 \Rightarrow b_i$ is sensitive resource

Tech Edge Co. Problem:

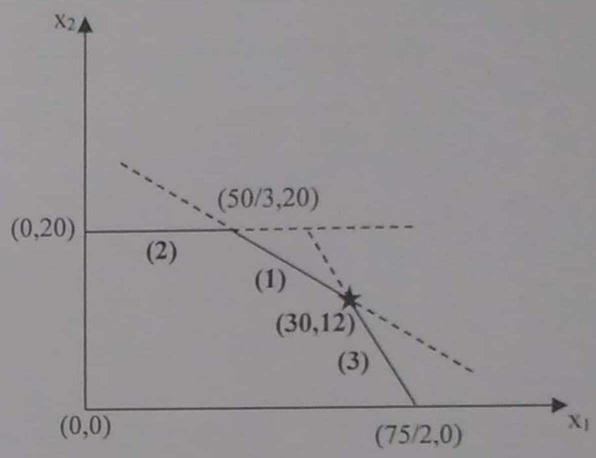
Max $Z = 50x_1 + 40x_2$

S.t. $3x_1 + 5x_2 \leq 150$ (Assembly time) **(1)**

$x_2 \leq 20$ (Special Display unit) **(2)**

$8x_1 + 5x_2 \leq 300$ (Warehouse Space) **(3)**

$x_1, x_2 \geq 0$



EFFECT OF CHANGE IN PROFIT COEFFICIENTS

For any c_j, the range of values for c_j over which the current optimal basis (solution) remains optimal, assuming no change in the other profit coefficients.

Allowable range for c_1 over which the current optimal basis remains optimal (keeping c_2 fixed)

Slope of constraint (3) ≤ Slope of objective function line ≤ Slope of constraint (1)

$-8/5 \leq -c_1/40 \leq -3/5$

$\Rightarrow -c_1/40 \leq -3/5 \Rightarrow c_1 \geq 24$ and $-8/5 \leq -c_1/40 \Rightarrow c_1 \leq 64$

$\Rightarrow \mathbf{24 \leq c_1 \leq 64}$

Allowable range for c_2 over which the current optimal basis remains optimal (keeping c_1 fixed)

Slope of constraint (3) ≤ Slope of objective function line ≤ Slope of constraint (1)

$-8/5 \leq -50/c_2 \leq -3/5$

$\Rightarrow \mathbf{125/4 \leq c_2 \leq 250/3}$

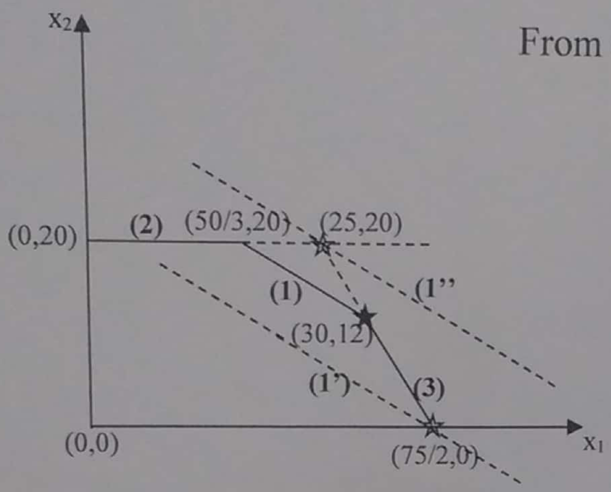
The values of the decision variables remain unchanged but the objective function value changes because change in profit coefficient.

EFFECT OF CHANGE IN RHS i.e. resource availability

For any b_i, the range of values for b_i over which the current basis remains feasible (with adjusted values for the basic variables), assuming no change in the other right-hand sides. Also called the feasibility range in which shadow price remains valid.

Range for b_1

(The values of the decision variables and the objective function value change)



From (1'): $3x_1+5x_2 \leq 3x75/2+5x0=112.5$

From (1''): $3x_1+5x_2 \leq 3x25+5x20=175$

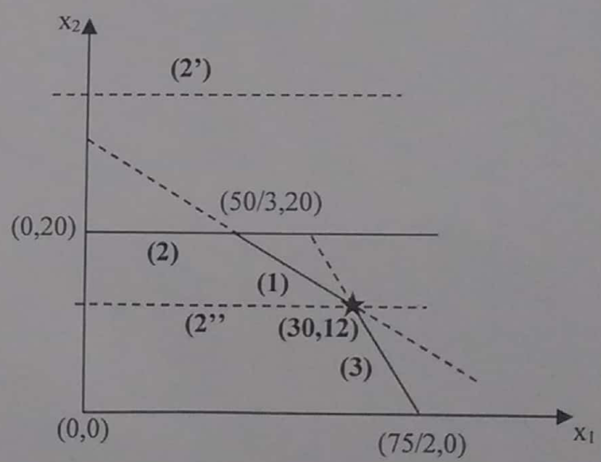
$112.5 \leq b_1 \leq 175$

Range for b_2:

From (2'): $x_2 \leq 12$

From (2''): $x_2 < \infty$

$12 \leq b_2 < \infty$



Range for b_3:

From (3'): $8x_1+5x_2 \leq 8x50/3+5x20=233.33$

From (3''): $8x_1+5x_2 \leq 8x50+5x0=400$

$233.33 \leq b_3 \leq 400$