Solution Class test 2

1

From the given set of final equations:

BV	x_1	x_2	x_3	x_4	x_5	RHS
x_3	$1/2$	0	1	$3/2$	$-1/2$	25
x_2	$1/2$	1	0	$-1/2$	$1/2$	25
z_j-c_j	3	0	0	4	1	300

(a)

Since x_1 is non-basic variable, so only z_1-c_1 will change.

$$z_1-c_1 = \underline{c_B B^{-1}} a_1 - c_1 = [4 \;\; 1]\begin{bmatrix}1\\2\end{bmatrix} - c_1 = 6-c_1 \geq 0 \Rightarrow \boxed{c_1 \leq 6}$$

Thus, if $c_1 \leq 6$, the current basis remains optimal.

(b)

$$\underset{\sim}{B^{-1}}\underset{\sim}{b'} = \begin{bmatrix}3/2 & -1/2\\ -1/2 & 1/2\end{bmatrix}\begin{bmatrix}b_1\\100\end{bmatrix} = \begin{bmatrix}\frac{3b_1}{2}-50\\ -\frac{b_1}{2}+50\end{bmatrix}$$

The current basis remains optimal if.

$$\left.\begin{aligned}\frac{3b_1}{2}-50 \geq 0 &\Rightarrow b_1 \geq \frac{100}{3}\\ \text{and } -\frac{b_1}{2}+50 \geq 0 &\Rightarrow b_1 \leq 100\end{aligned}\right\} \Rightarrow \boxed{\frac{100}{3} \leq b_1 \leq 100}$$

(c)

For $b_1 = 30$, the current basis would not be ~~optimal~~ feasible

To find new solution using dual simplex method.

BV	x_1	x_2	x_3	x_4	$x_5 \downarrow$	RHS
x_3	$1/2$	0	1	$3/2$	$-1/2$	$\cancel{25}$ $-5 \rightarrow$
x_2	$1/2$	1	0	$-1/2$	$1/2$	35
z_j-c_j	3	0	0	4	1	$\cancel{300}$ 220
Ratio	—	—	—	—	$1/(-\frac{1}{2}) = -2$	
x_5	-1	0	-2	-3	1	10
x_2	1	1	1	1	0	30
z_j-c_j	4	0	2	7	0	210

$$C_B B^{-1} b = [5 \;\; 7]\begin{bmatrix}-5\\35\end{bmatrix} = 220$$

$$\boxed{\text{optimal sol}^n.\ x_1 = 0,\ x_2 = 30,\ x_3 = 0 \qquad \underline{Z = 210}}$$

(d)

Let x_6 be the number of type 4 candy bars to be produced

$$z_6 - c_6 = \underline{c_B B^{-1}} a_6 - c_6 = [4 \; 1]\begin{bmatrix}3\\4\end{bmatrix} - 17 = -1$$

Hence, the current basis is not optimal.

BV	x_1	x_2	x_3	x_4	x_5	x_6 ↓	RHS	Ratio
x_3	$1/2$	0	1	$3/2$	$-1/2$	$5/2$	25	10 →
x_2	$1/2$	1	0	$-1/2$	$1/2$	$1/2$	25	50
$z_j - c_j$	3	0	0	4	1	-1	300	
x_6	$1/5$	0	$2/5$	$3/5$	$-1/5$	1	10	
x_2	$+2/5$	1	$-1/5$	$-4/5$	$3/5$	0	20	
$z_j - c_j$	$16/5$	0	$2/5$	$23/5$	$4/5$	0	310	

$$B^{-1}a_6 = \begin{bmatrix}3/2 & -1/2\\ -1/2 & 1/2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}5/2\\ 1/2\end{bmatrix}$$

$$\boxed{\text{Optimal sol}^n: \; x_2 = 20,\; x_1 = 0,\; x_3 = 0,\; x_6 = 10,\; Z = 310}$$

(2) Initial BFS using VAM

	Delhi	Mumbai	Kol	Chennai	Supply	Penalty	
Mysore	20	6 (2000)	17	5 (3000)	~~5000~~/2000	1	(11)
Nasik	12 (3000)	2 (1000)	18 (1000)	16	~~5000~~/~~4000~~/1000	~~10~~	10
Dummy	0	0	0 (2000)	0	2000	0	
Demand	3000	~~3000~~ 1000	~~3000~~ 1000	2000			
Penalty	12	2	(17)	5			
	8	4	1	(11)			

Optimality Test

				u_i
20 $\boxed{-4}$	6 (2000) $-\theta$	17 $+\theta$ $\boxed{5}$	5 (3000)	0
12 (3000)	2 $+\theta$ (1000)	18 $-\theta$ (1000)	16 $\boxed{-15}$	-4
0 $\boxed{-6}$	0 $\boxed{-16}$	0 (2000)	0 $\boxed{-17}$	-22
v_j → 16	6	22	5	

$\theta = \min\{1000, 2000\} = 1000$

				u_i
20 $\boxed{-4}$	6 (1000)	17 (1000)	5 (3000)	0
12 (3000)	2 (2000)	18 $\boxed{-5}$	16 $\boxed{-15}$	-4
0 $\boxed{-1}$	0 $\boxed{-11}$	0 (2000)	0 $\boxed{-12}$	-17
v_j: 16	6	17	5	

Since for All NBV cell $u_i+v_j-c_{ij} \le 0$ optimal sol^n.

Udlcote will have short supply of $\boxed{2000}$

Total cost $= 6\times1000 + 17\times1000 + 5\times3000 + 12\times3000 + 2\times2000$

$= \boxed{78000}$