Department of Industrial & Systems Engineering
Indian Institute of Technology Kharagpur

End-Semester Examination – Spring 2016

Operations Research (IM41082)

Time: 3 Hrs (Attempt **ALL** questions) **Full Mark: 50**

Note: State your assumptions clearly

Question 1 [2+1+3+3+1]

CU-Miner, a copper manufacturer, produces three different types of product: wire (A), disc (B) and bar (C). Two different raw materials are required for the production R1 (ore) and R2 (cleaner). Each kg production of A, B and C requires 1.4, 1.5 and 1.6 kg of R1, respectively. Raw material R2 requirement is 1 kg, 2 kg and 3 kg for each kg of products A, B, and C, respectively. Total available R1 and R2 are 2000 kg and 900 kg, respectively. Two operations are required Op1 and Op2 on two different machines for the final products. Time needed/kg and total available time (in hours) are given in the table below:

	A	B	C	Available
Op 1	4	6	10	3600
Op2	3	4	7	3000

The selling price per kg for product A is ₹ 100, and for B and C are ₹ 90.

(a) Write the LP formulation and solve using the concept of revised simplex (Matrix form).

(b) Is there any redundant constraint in the problem? If yes, identify and explain.

(c) Another home appliance company HOMEDIVINE wants to buy products A and B only. What should be the minimum price he should offer to CU-Miner such that it will produce only A and B and sell to HOMEDIVE? (Assume only Integer price)

(d) A Cu- pots maker POTTERY is interested in buying products B and C only. How much minimum price he should offer to B and C such that Cu-Miner will only manufacture B and C? (Assume only Integer price)

(e) Which of the offers (out of c and d) will be lucrative for Cu-Miner?

Question 2 [6+4]

(a) Consider the following LP

Maximize $Z = 8x_1 - 9x_2 + 12x_3 + 4x_4 + 11x_5$

Subject to

$2x_1 - 3x_2 + 4x_3 + x_4 + 3x_5 \le 5$

$x_1 + 7x_2 + 3x_3 - 2x_4 + x_5 \le 3$

$5x_1 + 4x_2 - 6x_3 + 2x_4 + 3x_5 \le 16$

$x_i \ge 0, i = 1, 2, \ldots, 5$

Use duality and complementary slackness theorem to find if (0, 2, 1, 7, 0) is the optimal solution.

(b) The setup time for each job on various machines is given by the following table. Find an optimal assignment of jobs to machines such that the total setup time is minimized. Has the problem multiple optimal solutions? Justify.

		Machine		
		1	**2**	**3**
Job	**A**	3	10	15
	B	15	20	23
	C	15	20	23
	D	21	25	27

Question 3 (Assume GD/∞/∞) [4 +(3+3)]

(a) A telephone company, TELNET is planning to build new telephone boxes for a city. Initial survey shows that approximately 100 persons per hour wants to make a phone call. Duration of the call is approximately exponentially distributed with mean 1 minute. How many (minimum) such boxes is needed such that mean waiting time is less than 15 seconds?

(b) A computer consists of three processors and their main task (priority) is to execute jobs from users. These jobs arrive according to the Poisson process with average rate 20 jobs per minute. The execution time at each processor is exponentially distributed with mean 2 seconds. When any processor becomes idle (no queue) it starts executing jobs from server till any job from user arrives. Assume server jobs are always available and it takes 5 seconds to finish a server job. Also assume that unfinished server jobs can be stored in the memory and can be resumed later, i.e. from the point where it was interrupted.

(i) Find how many server jobs will be completed in 30 minutes.

(ii) How many more server jobs will be finished in 30 minutes if processors are replaced, which takes 1 second.

Question 4 **[2+2+3+3]**

A manufacturing company produces three products (A, B, and C) using labor and material with the objective of maximizing the total profit to determine the optimal production plan. The problem is formulated as the following linear program.

Maximize $Z = 3x_1 + x_2 + 5x_3$

Subject to:

$6x_1 + 3x_2 + 5x_3 \leq 45$ (Labor)

$3x_1 + 4x_2 + 5x_3 \leq 30$ (Material)

$x_1, x_2, x_3 \geq 0$

where x_1, x_2, and x_3 are the amount of products A, B, and C. Let x_4 and x_5 be slack variables for labor and material constraints, respectively. The optimal solution is given by the following tableau.

Basis	x_1	x_2	x_3	x_4	x_5	RHS
x_1	1	-1/3	0	1/3	-1/3	5
x_3	0	1	1	-1/5	2/5	3
$z_j - c_j$	0	3	0	0	1	30

Using the procedure of sensitivity analysis, answer the following questions

(a) What should be the unit profit of Product B before it becomes worthwhile to manufacture?

(b) Suppose an additional 15 units of material may be obtained at a cost of \$10. Is it profitable to do so?

(c) Find the optimal solution when the available material is increased to 60 units.

(d) Suppose a "supervision" constraint, $x_1 + x_2 + 3x_3 \leq 18$ is added to the original problem, find the new optimal production plan.

Question 5 **[(1+2+3)+4]**

(a) A hospital needs to purchase 3 gallons of a perishable medicine for use during the current month and 4 gallons for use during the next month. Because the medicine is perishable, it can only be used during the month of purchase. Two companies (X and Y) sell the medicine. The medicine is in short supply. Thus, during the next two months, the hospital is limited to buying at most 5 gallons from each company. The companies charge the prices shown in the following table.

Company	Current month's price per gallon (\$)	Next month's price per gallon (\$)
X	800	720
Y	710	750

(i) Formulate a balanced transportation model in tabular form to minimize the cost of purchasing the needed medicine.

(ii) Use Vogel's method to find a basic feasible solution for the problem

(iii) Use the transportation simplex to find the optimal buying strategy for the hospital.

(b) Consider the problem in which two factories supply three stores with a commodity. The number of supply units available at source 1 and 2 are 200 and 300 units respectively. The demand at stores 1, 2, and 3 are 100, 200 and 50 only. The units may be transshipped among the factories and the stores before reaching the final destination. Set up the transportation table based on the following units costs.

		Factory		**Store**		
		1	2	1	2	3
Factory	1	0	6	7	8	9
	2	6	0	5	4	3
Store	1	7	2	0	5	1
	2	1	5	1	0	4
	3	8	9	7	6	0