Department of Industrial & Systems Engineering
Indian Institute of Technology Kharagpur

End-Semester Examination – Spring 2016-17

Operations Research (IM41082)

Time: 3 Hrs (Attempt **ALL** questions) **Full Mark: 50**

Note: State your assumptions clearly

Question 1 [5+4+1]

A manufacturer produces three different types of product: A, B and bar C. Two different raw materials are required for the production R1 (ore) and R2 (cleaner). Each kg production of A, B and C requires 3, 2 and 1 kg of R1, respectively. Raw material R2 requirement is 5 kg, 3 kg and 1 kg for each kg of products A, B, and C, respectively. Total available R1 and R2 are 100 kg and 160 kg, respectively. The selling price per kg for product A is ₹ 50, B is ₹ 10 and C is ₹ 15.

(a) Write the LP formulation. Solve using the concept of revised simplex (Matrix form).

(b) Market research have shown that customers are buying either product A &B together or B & C together. What should be the price of B such that company is producing A &B only or A & C only. (Assume only Integer price)

(c) Which combination in part (b) will give the higher revenue?

Question 2 [(2+3)+5]

(a) Consider the following LP

Maximize $Z = 18x_1 - 7x_2 + 12x_3 + 5x_4 + 8x_6$

Subject to

$$2x_1 - 6x_2 + 2x_3 + 7x_4 + 3x_5 + 8x_6 \le 1$$
$$-3x_1 - x_2 + 4x_3 - 3x_4 + x_5 + 2x_6 \le -2$$
$$8x_1 - 3x_2 + 5x_3 - 2x_4 + 2x_6 \le 4$$
$$4x_1 + 8x_3 + 7x_4 - x_5 + 3x_6 \le 1$$
$$5x_1 + 2x_2 - 3x_3 + 6x_4 - 2x_5 - x_6 \le 5$$
$$x_j \ge 0, j = 1, 2, ..., 6$$

(i) Convert this problem into dual.

(ii) If (2, 4, 0, 0, 7, 0) is the optimal solution of the above problem, find the optimal solution of the dual problem using complementary slackness theorem.

(b) Consider the problem of assigning four machines (A, B, C, and D) to the five vacant positions (1, 2, 3, 4 and 5) in a machine shop. The cost of material handling is a function of the location of the machine and is shown in the following Table.

Machine	Weekly cost of material handling in thousands of Rupees for location of the machines at				
	Site 1	Site 2	Site 3	Site 4	Site 5
A	3	9	8	7	5
B	6	1	5	6	7
C	9	4	7	10	3
D	8	5	4	2	7

What will be the maximum possible cost if the machines are located totally arbitrarily?

Question 3 [(3+3)+4]

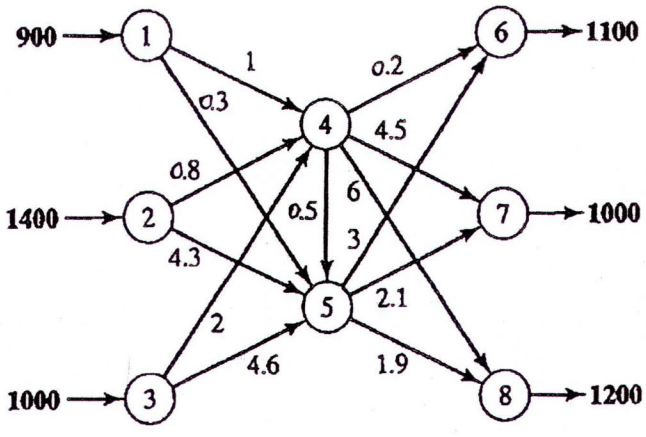
(a) In a 3x3 transportation problem, let x_{ij} be the amount shipped from source i to destination j and let c_{ij} be the corresponding transportation cost per unit. The amounts of supply at sources 1, 2, and 3 are 15, 30, and 85 units, respectively, and the demands at destinations 1, 2, and 3 are 20, 30, and 80 units, respectively. Assume that the starting northwest-corner solution is optimal and that the associated values of the dual variables are given as $u_1 = -2$, $u_2 = 3$, $u_3 = 5$, $v_1 = 2$, $v_2 = 5$, and $v_3 = 10$.

(i) Find the associated optimal cost.

(ii) Determine the smallest value of c_{ij} for each nonbasic variable that will maintain the optimality of the northwest-corner solution.

(b) The network in the adjacent figure shows the routes for shipping cars from three plants (nodes 1 to 3) to three dealers (nodes 6 to 8) by way of two distribution centers (nodes 4 and 5). The shipping costs per car

(in $100) are shown on the arcs. The numbers of supply units available at plants 1, 2, and 3 are 900, 1400, and 1000; those demanded at dealers 6, 7, and 8 are 1100, 1000, and 1200, respectively. Assuming that distribution center 4 can sell 240 cars directly to customers; formulate the problem as a balanced transportation problem in the tabular form.



Question 4 **[1.5+2+3+3.5]**

Consider the following LPP

$$\text{Maximize } Z = 2x_1 - x_2 + x_3$$

Subject to

$$3x_1 - 2x_2 + 2x_3 \le 15 \text{ (Resource 1)}$$
$$-x_1 + x_2 + x_3 \le 3 \text{ (Resource 2)}$$
$$x_1 - x_2 + x_3 \le 4 \text{ (Resource 3)}$$

and $x_1, x_2, x_3 \ge 0$.

If we let x_4, x_5, and x_6 be the slack variables for the respective resource constraints, the simplex method yields the following final set of equations:

$$Z + 2x_3 + x_4 + x_5 = 18$$
$$x_2 + 5x_3 + x_4 + 3x_5 = 24$$
$$2x_3 + x_5 + x_6 = 7$$
$$x_1 + 4x_3 + x_4 + 2x_5 = 21$$

(a) Find the shadow price of each resource.

(b) Find the range of resource 1 (i.e., b_1) in which the current solution remains feasible.

(c) Find the range for the cost coefficients c_1 and c_2 in which the current solution remains optimal.

(d) If a new constraint $2x_1 + x_2 + 3x_3 \le 60$ is introduced, apply dual simplex method if the current solution is not feasible/optimal.

Question 5 (Assume GD/∞/∞) **[3+5+2]**

(a) For M/M/1 model, if λ = 60/hour and μ = 80/hour find P_0, L, L_q, W and W_q.

(b) In an assembly plant, a robot is performing the assembly operation. The arrival rate of job is exponentially distributed with 120 jobs/hour. Robot's assembly execution is also exponentially distributed with service rate 80 jobs/hour. If the cost each robot is ₹20 /hour and waiting cost of jobs are ₹48 /hour/unit. Company is looking to increase the number of robots to either 2 or 3. Which option will be cost efficient?

(c) Write the condition in which the transportation problem has **(i)** Degenerate solution. **(ii)** Multiple optimal solutions.

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