Department of Industrial & Systems Engineering
Indian Institute of Technology Kharagpur

End-Semester Examination – Spring 2017-18

Operations Research (IM41082)

Time: 3 Hrs (Attempt ALL questions) **Full Mark: 50**

Note: Assume missing data suitably.

Question 1 [3+5+2]

Consider the following linear Programming

$$\text{Max } z = 5x_1 + 4x_2 + 3x_3$$

$$\text{s.t. } 2x_1 + 3x_2 + x_3 \le 5$$

$$4x_1 + x_2 + 2x_3 \le 11$$

$$3x_1 + 4x_2 + 2x_3 \le 8$$

$$x_1, x_2, x_3 \ge 0$$

Let x_4, x_5, x_6 be slack variables for the respective constraints. The final (optimal) simplex table for this LP is given by

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
x_1	0	1	2	0	2	0	-1	2
x_5	0	0	-5	0	-2	1	0	1
x_3	0	0	-1	1	-3	0	2	1
obj $(z_j - c_j)$	1	0	3	0	1	0	1	13

a) Within what range of cost coefficients of x_1, x_2 and x_3 the current solution remains optimal?

b) Using Dual simplex find the optimal solution if $\mathbf{b} = (5,11,8)^T$ is changed to $\mathbf{b'} = (18, 9, 9)^T$.

c) Will the objective function value change by adding a constraint $3x_1 + 4x_2 + 2x_3 \le 10$ in the original problem? Explain.

Question 2 [4+6]

In a large machine repairing company, technicians use to get their tools from the tool centre, which is managed by a single manager. Inter-arrival time of technicians and the processing time of manager are exponentially distributed. The mean number of technicians asking for tools is 5 per hour, and the average time taken to handle one request for tools be 10 minutes by the manager. The manager is paid ₹ 50 per hour and each technician is paid ₹ 80 per hour.

a) From the model, determine the average number of workers waiting for tools at the tool centre (including the worker being served). Use this to calculate the average cost per hour of providing tools. This will include the cost of time lost by the workers, and the cost of employing the tool distributor.

b) Repeat your analysis, supposing that there are now two managers (each with same processing rate and cost) and see if employing a second manager is justified.

Question 3 [6+4]

a) In the following LP problem

$$\text{Max } z = 3x_1 + 2x_2 + 5x_3$$

$$\text{s.t. } x_1 + 2x_2 + x_3 \le 43$$

$$x_1 + 4x_2 \le 42$$

$$3x_1 + 2x_3 \le 46$$

$$x_1, x_2, x_3 \ge 0$$

Using complementary slackness theorem find if (0, 10, 23) is the optimal primal solution.

b) Convert the following problem into its dual:

Minimize $Z = 5x_1 + 7x_2 + 3x_3$

Subject to

$2x_1 + 3x_2 + 6x_3 = 10$

$5x_1 + 3x_2 + 5x_3 \geq 8$

$6x_1 + 4x_2 + 3x_3 \leq 9$

$3x_1 + 7x_2 + 5x_3 \geq 6$

$x_1 \geq 0$, x_2 – unrestricted, and $x_3 \leq 0$.

Question 4 **[3+5+2]**

The cost of sending 1 million KWH of power from the different plant of NTPC to the different city is listed in the following table.

From	To				Supply (million KWH)
	City 1	City 2	City 3	City 4	
Plant 1	8	6	10	9	**35**
Plant 2	9	12	13	7	**50**
Plant 3	14	9	16	5	**40**
Demand (million KWH)	**45**	**20**	**30**	**30**	

a) Find the basic feasible solution using Vogel's Approximation method.

b) If the basic feasible solution obtained in part (a) is not optimal, determine the optimal cost of power transmission.

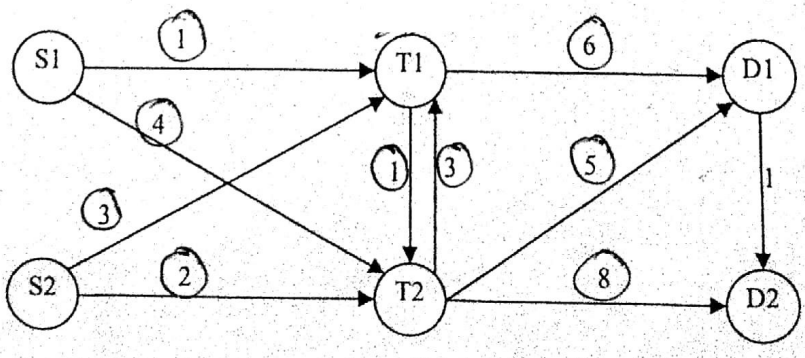
c) Formulate the dual problem in terms of the given values.

Question 5 **[5+5]**

a) To stimulate interest and provide an atmosphere for intellectual discussion, a faculty decides to hold special seminars on four contemporary topics: Sustainability, Energy, Transportation, and Automation. Such Seminars should be held every week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows. Find an optimal schedule of seminars.

	Sustainability	Energy	Transportation	Automation
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

b) Consider the following transportation network. Numbers associated with links are the cost of transportation per unit. The number of supply units available at S1, S2 and T1 are 100, 100 and 250 units, respectively. The demand at D1 and D2 are 200 and 300, respectively. Formulate an equivalent transportation problem in the tabular form.



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