Advanced Fluid Mechanics (ME61003) / Fluid Mechanics (ME60011)

Class Test-1, September 2019, IIT Kharagpur

Full Marks: 60, Time: 1 hour

All questions are compulsory. Each question carries 20 marks

Q1. State whether the following statements are true or false with proper justification.

(a) Streamlines, streaklines and pathlines are identical for an inviscid flow.
(b) An incompressible flow is necessarily a constant density flow.
(c) Circulation around any closed contour in an irrotational flow field is always zero.
(d) Streamlines do not exist for flow fields for which stream function cannot be defined.
(e) There is no pressure variation normal to straight streamlines in an inviscid flow.

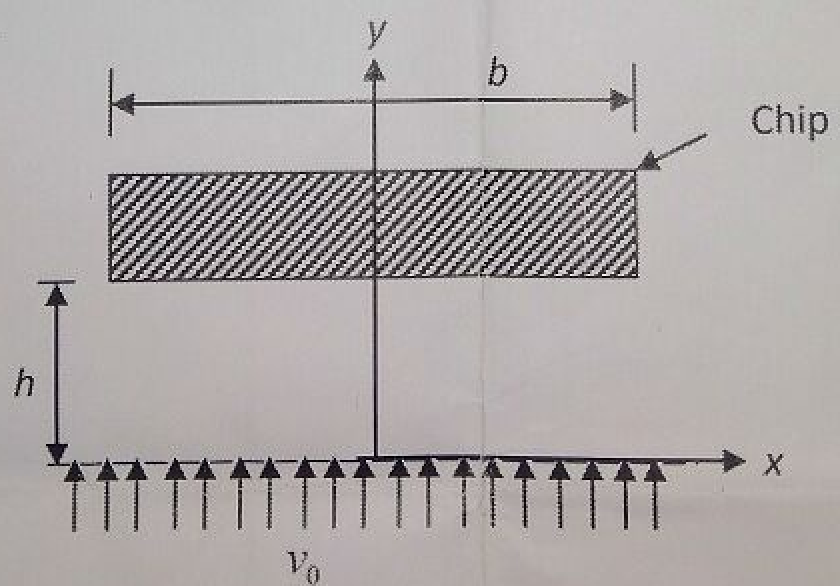
Q2. The three components of velocity in a velocity field are given by

$$u = Ax + By + Cz,\ \ v = Dx + Ey + Fz, w = Gx + Hy + Kz.$$

Determine the relationship/s among the coefficients A through K necessary for the following cases:

(a) If this is to be a possible incompressible flow field.
(b) If this is to be an irrotational flow field.
(c) If this flow field has to represent a rigid body motion.

Q3. A heated rectangular electronic chip floats on the top of a thin layer of air, above a bottom plate as shown in figure below. Air of density ρ, is blown at a uniform velocity v_0 through the holes in the bottom plate. The chip has width w perpendicular to the plane of the diagram. There is no flow in the z direction. Assume a steady, inviscid, constant density flow in the gap between the chip and the bottom plate. In addition assume the flow in the x direction in the gap under the chip to be uniform.



a) Obtain an expression for the velocity in the x direction.
b) Obtain an expression for the velocity in the y direction.
c) Find a general expression for the acceleration of a fluid particle in the gap in terms of v_0, h, x and y.
d) If static pressure at the origin is p_0, obtain an expression for the net pressure force exerted by the air on the bottom surface of the chip in terms of p_0, v_0, ρ, h, b, w.

Advanced Fluid Mechanics (ME61003) / Fluid Mechanics (ME60011)

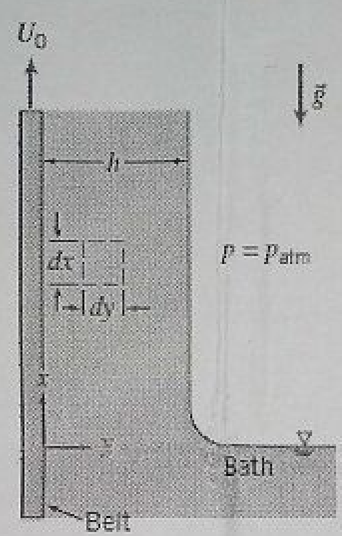
Class Test-2, November 2019, IIT Kharagpur

Full Marks: 60, Time: 1 hour

All questions are compulsory. Each question carries 30 marks

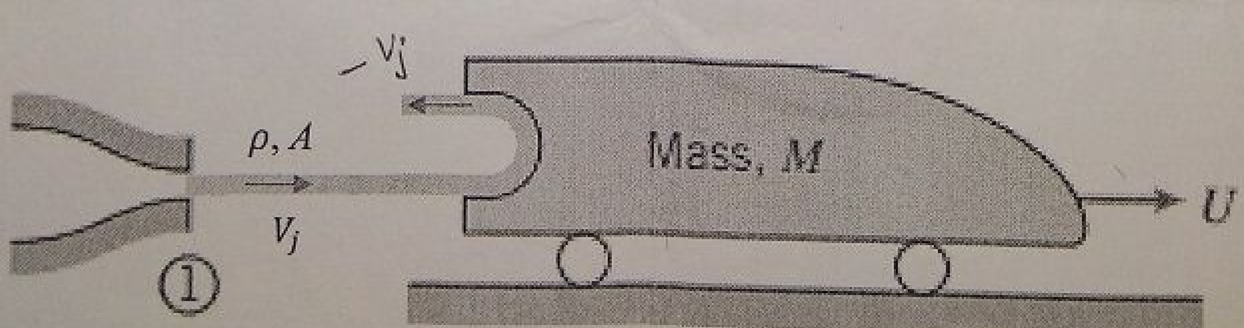
Q1. A continuous belt, passing upward through a chemical bath at speed U_0, picks up a liquid film of thickness h, density ρ, and viscosity μ. Gravity tends to make the liquid drain down, but the movement of the belt keeps the liquid from running off completely. Assume that the flow is fully developed and laminar with zero pressure gradient, and that the atmosphere produces no shear stress at the outer surface of the film.

(a) State clearly the boundary conditions to be satisfied by the velocity at $y = 0$ and $y = h$.
(b) Obtain an expression for the velocity profile.
(c) What is the velocity U_0 for which there is no net flow either up or down.



Q2. Starting from rest at $t = 0$, the cart shown is propelled by a hydraulic catapult (liquid jet). The jet strikes the curved surface and makes a 180° turn, leaving horizontally. Rolling resistance may be neglected, but an aerodynamic drag force acts on the cart proportional to the square of its speed, $F_D = kU^2$. The mass of the cart is M and the jet of water (of constant area A) leaves the nozzle with a speed V_j (relative to the ground).

(a) Derive an expression for the cart acceleration as a function of cart speed, $U(t)$ and other given parameters. For your analysis, you may neglect the mass of the liquid in contact with the cart in compassion to the cart mass.
(b) What is the terminal speed of the cart?



Advanced Fluid Mechanics (ME61003)/ Fluid Mechanics (ME60011),

Mid Semester Examination, September 2019, IIT Kharagpur,

Full Marks = 120, Time: 2 hours

All questions are compulsory. Each question carries 30 marks

Q1. Consider a two-dimensional velocity field:

$$\vec{V} = u(x,y,t)\hat{\imath} + v(x,y,t)\hat{\jmath}$$

a) What is the relation between $u, v, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ for a stream function to be defined for the above flow field?

b) What is the relation between $u, v, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ for a velocity potential to be defined for the above flow field?

c) What are the relations between $u, v, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ for both stream function and velocity potential to be defined for the above flow field?

d) In case if a stream function, ψ can be defined, show that lines of constant ψ are the flow streamlines.

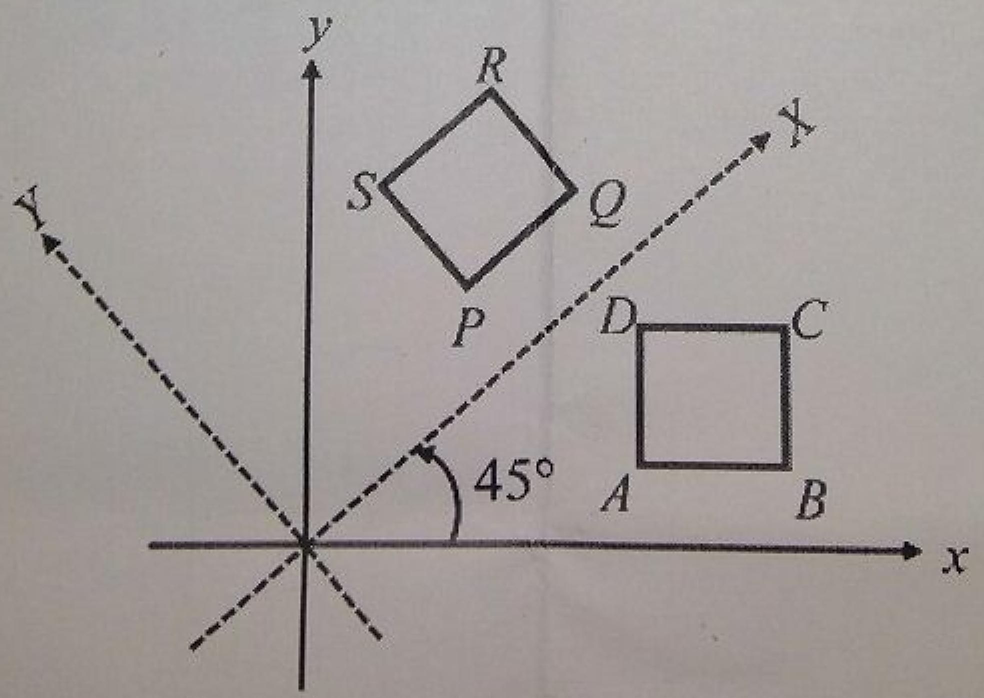
e) In case if both the stream function and the velocity potential can be defined, show that the velocity potential ϕ satisfies the Laplace equation.

f) In case if both the stream function and velocity potential can be defined, show that lines of constant ϕ are everywhere perpendicular to line of constant ψ except at the stagnation points.

Q2. A steady, two-dimensional flow field is represented by the stream function:

$$\psi = Kxy$$

where $K = 1\ s^{-1}$. Consider a new coordinate system (labelled as X-Y) obtained by rotating the x and y axes counter clockwise through an angle 45° with respect to the original coordinate system keeping the origin fixed as shown in the figure below. Consider two rectangular fluid elements $ABCD$ and $PQRS$ in the flow field as shown in the figure. $ABCD$ has its adjacent sides parallel to the x and y axes whereas the fluid element $PQRS$ has its adjacent sides parallel to the X and Y axes.



a) Obtain an expression for the vorticity vector $\vec{\Omega}$ for the above flow field.

b) Obtain an expression for the volumetric strain rate of a fluid element located at any point (x,y).

c) Obtain an expression for the stream function for the above flow field expressed in terms of the coordinates X and Y measured with respect to the new coordinate system.

d) Does the fluid element $ABCD$ undergo any linear deformation? If yes, sketch its deformation qualitatively.

e) Does the fluid element $ABCD$ undergo any angular deformation? If yes, sketch its deformation qualitatively.

f) Does the fluid element $ABCD$ undergo any rotation?

g) Does the fluid element $PQRS$ undergo any linear deformation? If yes, sketch its deformation qualitatively.

h) Does the fluid element $PQRS$ undergo any angular deformation? If yes, sketch its deformation qualitatively.

i) Does the fluid element $PQRS$ undergo any rotation?

Q3. The velocity components in a two-dimensional, inviscid, constant density (=1000 kg/m^3), steady flow field are given as follows: $u = A(x+y), v = -A(x+y)$, where A=1 s^{-1}. Consider a directed line segment in the flow field, connecting the points P(0,0) and Q(3,3). The pressure is given as zero gauge at the origin. The acceleration due to gravity is g =10 m/s^2 and it acts along the negative z direction.

(a) Is the line $\overrightarrow{PQ}$ a streamline? Justify with calculations.

(b) Can the Bernoulli's equation be applied to find the change in pressure experienced on moving from the point P to the point Q along the direction $\overrightarrow{PQ}$? (Merely stating yes or no without proper mathematical justification will not carry any marks)

(c) Starting from Euler's equation of motion in differential form, find the pressure at Q.

Note: Euler's equation of motion in differential form for a steady flow is given as

$$-\frac{\nabla p}{\rho} + \vec{b} = (\vec{V} \cdot \nabla)\vec{V}$$

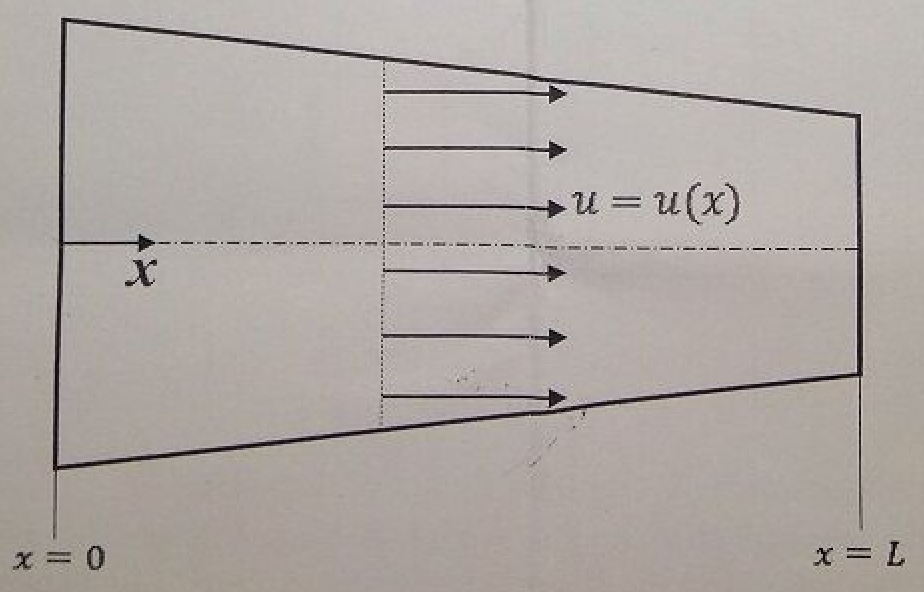
where $\vec{b}$ is the body force per unit mass. You may use the following vector identity for your analysis:

$$(\vec{V} \cdot \nabla)\vec{V} = \frac{1}{2}\nabla(\vec{V} \cdot \vec{V}) - \vec{V} \times (\nabla \times \vec{V})$$

Q4. A fluid of constant density ρ resides in a horizontal nozzle of length L having a cross-sectional area that varies smoothly between A_i and A_0 via:

$$A(x) = A_i + (A_o - A_i)\frac{x}{L}$$

Here the x-axis lies on the nozzle's centreline and $x = 0$ and $x = L$ are the horizontal locations of nozzle's inlet and outlet, respectively. The initial pressure inside the nozzle is p_0. At $t = 0$, the pressure at the inlet is raised to $p_i > p_o$ and the outlet pressure is maintained at p_0. As a result, the fluid begins to flow horizontally through the nozzle. Assume the flow inside the nozzle to be inviscid and the velocity field to be one-dimensional i.e. $u = u(x)$ and $v \approx 0$.



Derive a differential equation for the time-dependent volume flow rate $Q(t)$, through the nozzle from the unsteady Bernoulli equation. Your answer should look like:

$$\frac{dQ}{dt} = f(Q, A_i, A_o, L, p_i, p_o, \rho)$$

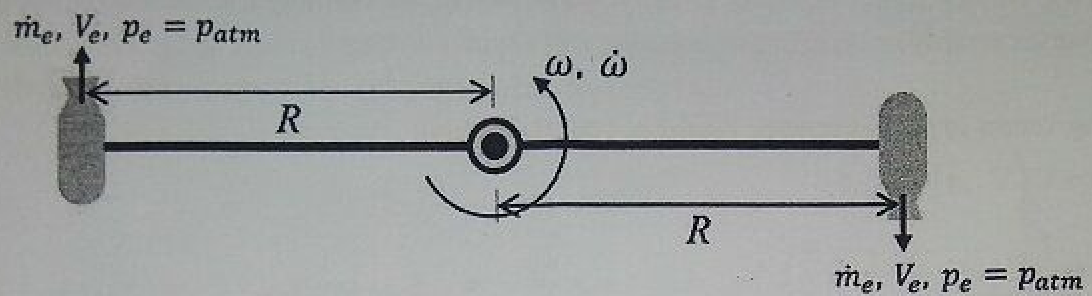
You need not solve this differential equation.

Advanced Fluid Mechanics (ME61003) / Fluid Mechanics (ME60011)

End Autumn Semester Examination, November 2019, IIT Kharagpur

Full Marks: 100, Time: 3 hours

Q1. Two small identical rockets are attached to the opposite ends of a rigid rod pivoted at the origin as shown in the figure below. The pivot is frictionless and the rod is free to rotate in the horizontal plane about the origin. The initial mass of each rocket is M_0 and it starts from rest upon ignition at time $t = 0$. Exhaust gases leave the rocket at a constant mass flow rate of $\dot{m}_e$ and with a constant and uniform velocity V_e relative to the rocket. The pressure at the nozzle exit is atmospheric pressure. Derive an expression for the angular velocity $\omega(t)$ of the rod. For your analysis you may assume that the unburned fuel and rocket structures have zero momentum relative to the rocket. Also neglect gravity, air drag, and the rod mass. **(15 marks)**



Q2.

(a) Starting with the Reynolds transport theorem for balance of linear momentum applied to a stationary control volume of fixed shape in a fluid flow, derive the differential form of linear momentum equation for a fluid flow (Note: Leave your answer in terms of the stress tensor components, τ_{ij}).

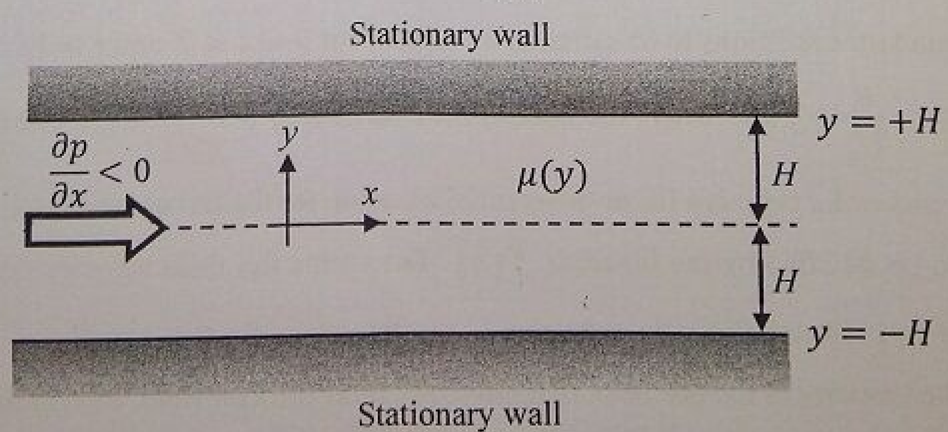
(b) For isotropic Newtonian fluid, the components of the stress tensor are given by the constitutive relationship:

$$\tau_{ij} = -p\delta_{ij} + \lambda\left(\nabla\cdot\vec{V}\right)\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where μ and λ are functions of position. Combing this constitutive equation with the result of part (a), derive the Naiver Stokes equations for incompressible flow of a Newtonian fluid. Do not assume the fluid viscosity to be a constant in your derivation. **(7+8=15 marks)**

Q3. Consider steady fully developed laminar incompressible flow of a Newtonian fluid in the gap between two large, smooth parallel plates that are both stationary. The upper and lower walls are located at $y = \pm H$, respectively as shown in the figure below. An additive in the liquid causes its viscosity to vary in the y-direction as $\mu(y) = \mu_0\left(1 + \frac{\gamma y^2}{H^2}\right)$.

Here the flow is driven by a constant nonzero pressure gradient: $-\frac{\partial p}{\partial x}$ = constant.



(a) Starting with the Navier Stokes equations derived in part (b) of question 2 coupled with the incompressibility constraint, derive an expression for $u(y)$ when $-1 < \gamma < 0$.

(b) What shear stress is felt on the lower wall? Explain why your answer does not depend on the value of γ.

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(c) If $-1<\gamma<0$, will the volume flux be higher or lower than the case when $\gamma=0$? Give an answer from a physical viewpoint without any mathematical calculations.

(10+5+5=20 marks)

Q4.

(a) What is Kolmogorov length scale? Derive an expression for the same in terms of the kinematic viscosity and the rate of dissipation of turbulent kinetic energy.

(b) Staring with the Navier Stokes equation in vector form, derive the vorticity transport equation in vector form. Identify the vortex stretching term in this equation and explain its consequence in turbulent flow physically.

(5+8=13 marks)

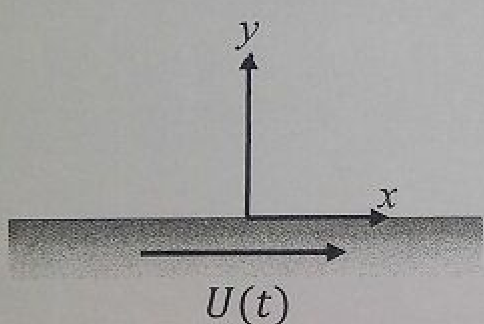
The following vector identifies may be useful in your derivation:

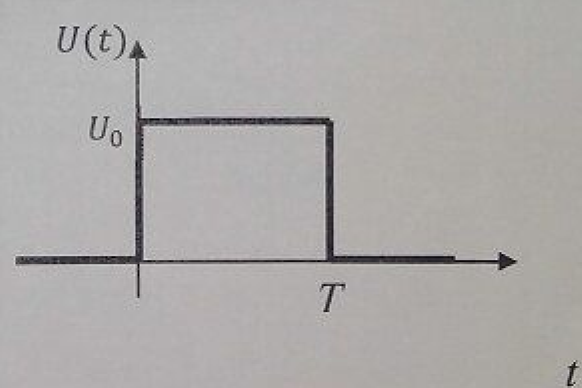
$$\nabla\times\left(\nabla\times\vec{A}\right)=\nabla\left(\nabla\cdot\vec{A}\right)-\nabla^2\vec{A}$$

$$\vec{A}\times\left(\nabla\times\vec{A}\right)=\frac{1}{2}\nabla\left(\vec{A}\cdot\vec{A}\right)-\left(\vec{A}\cdot\nabla\right)\vec{A}$$

$$\nabla\times\left(\vec{A}\times\vec{B}\right)=\vec{A}\left(\nabla\cdot\vec{B}\right)-\vec{B}\left(\nabla\cdot\vec{A}\right)+\left(\vec{B}\cdot\nabla\right)\vec{A}-\left(\vec{A}\cdot\nabla\right)\vec{B}$$

Q5. An infinite flat plate is located at $y=0$ (xz-plane) below an initially stationary constant property Newtonian fluid of density ρ and viscosity μ occupying the upper half space, $y>0$ as shown in the figure below. The plate is stationary until $t=0$ when it suddenly starts translating in its own plane in the positive x-direction at a constant speed U_0. This motion continues until $t=T$ when the plate suddenly stops moving. There are no externally applied pressure gradients. Ignore the effects of any body forces.





(a) Starting with the x-component of momentum equation, determine a linear partial differential equation for $u(y,t)$.

(b) State the boundary conditions to be satisfied by $u(y,t)$ for $0<t<T$ and $t>T$.

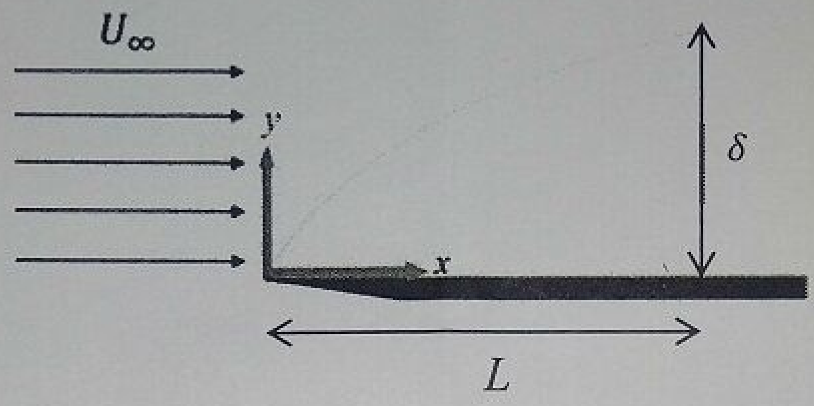
(c) For $0<t<T$, $\dfrac{u}{U_0}=f(\eta)$ where the independent variable $\eta=\dfrac{y}{g(t)}$. Determine the function $g(t)$.

(d) Derive a second-order ordinary linear differential equation for the unknown function $f(\eta)$. State the boundary conditions to be satisfied by the function $f(\eta)$. Determine the fluid velocity field, $u(y,t)$ for $0<t<T$ by solving for $f(\eta)$. Sketch the expected velocity profile shapes for $0<t<T$.

(e) Exploiting the linearity of the problem, determine the fluid velocity field, $u(y,t)$ for $t>T$. Sketch the expected velocity profile shapes for $t>T$ (Hint: the prescribed wall velocity function $U(t)$ could be expressed as the superposition of two step functions).

(2+2+3+5+10=22 marks)

Q6. Consider laminar boundary layer flow of Newtonian fluid of density ρ and dynamic viscosity μ over a stationary flat plate as shown in the figure below. The plate length is L and its width perpendicular to the plane of paper is b. The velocity of the uniform free steam is U_∞. Let δ denote the boundary layer thickness at a distance L from the leading edge.



Analytical solution of the boundary layer equations yields the relation:

$$\frac{\delta}{L} = c_1 \, Re_L^n$$

where Re_L is the Reynolds number defined as $Re_L = \frac{\rho U_\infty L}{\mu}$ and c,n are constants.

(a) Write the boundary layer equations for flow over a flat plate in their most simplified form.
(b) Determine the value of n by means of an order-of-magnitude analysis of the boundary layer equations.
(c) The drag force on the plate is traditionally expressed in terms of the non-dimensional drag coefficient, C_D defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho U_\infty^2 L b} = c_2 Re_L^m$$

where F_D is the total drag force acting on the plate up to length L. Determine the value of m by means of an order-of-magnitude analysis.

(2+7+6=15 marks)