INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: 20-02-2018 AN **Time: 2 Hours** **Full Marks: 60**
No. of Students: 70 **Mid Spring Semester Examination 2018**
Sub. No.: ME60014/ME61004 **Sub. Name: Convective Heat and Mass Transfer**

Instructions: Attempt all questions. Symbols have their usual meanings. Please explain your work carefully. Make suitable assumptions wherever necessary. Please state your assumptions clearly. **Clearly indicate the coordinate system used in your analysis.** The following information may be useful:

$\int_B b_j n_j \, dA = \int_R \frac{\partial b_j}{\partial x_j} dV$, where **b** is a continuous vector field in a region R bounded by a surface B.

Q1. (a) Starting with Cauchy's equations of motion, $\rho \frac{Dv_i}{Dt} = \rho g_i + \frac{\partial \tau_{ji}}{\partial x_j}$, show that

$$\rho \frac{D}{Dt}\left(e_{kin} + e_{pot}\right) = \dot{w}_s - \dot{w}_c - \Phi, \qquad (1)$$

where ρ is the fluid density, $e_{kin} = \frac{1}{2} v_i v_i$, $e_{pot} = -g_i x_i$, v_i is the component of the fluid velocity, **v**, in the x_i-direction, g_i is the component of the constant gravitational acceleration, $\mathbf{g}$, in the direction of the coordinate axis x_i, $\dot{w}_s = \frac{\partial}{\partial x_j}\left(v_i \tau_{ji}\right)$, $\dot{w}_c = -p\nabla\cdot\mathbf{v}$, $\Phi = \pi_{ji} d_{ij}$, τ_{ij}, π_{ij} and d_{ij} are respectively the elements of the stress tensor, the deviatoric stress tensor and the deformation tensor, p is the thermodynamic pressure, $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j}$ denotes material derivative and t is the time.

(b) The 'internal energy form' of the local balance of thermal energy may be expressed as

$$\rho \frac{De}{Dt} = -\nabla\cdot\mathbf{q} + \dot{w}_c + \Phi, \qquad (2)$$

where e is the specific internal energy and $\mathbf{q}$ is the heat flux vector. Using equation (2) and the Gibbs equation, $Tds = de + pdv$, for a simple compressible substance, where T is the absolute temperature, s is the specific entropy and $v = \frac{1}{\rho}$ is the specific volume, derive the following 'entropy form' of the local balance of thermal energy: $\rho T \frac{Ds}{Dt} = \Phi - \nabla\cdot\mathbf{q}$.

(c) Starting with the 'entropy form' of the local balance of thermal energy, stated in part (b), use the thermodynamic relations, $\left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$ and $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$, to show that the energy equation for a 'Fourier' fluid may be expressed in 'c_p form' as

$$\rho c_p \frac{DT}{Dt} = \nabla\cdot(k\nabla T) + \Phi + \beta T \frac{Dp}{Dt},$$

where c_p, k and β are respectively the specific heat capacity at constant pressure, the thermal conductivity and the coefficient of thermal expansion of the fluid.

(d) Which of the terms, $\dot{w}_c$ and Φ, in equations (1) and (2), if any, result in a change in the total energy (sum of internal, kinetic and potential energies) of the system ? Give reasons to justify your answer.

(e) Which of the terms, $\dot{w}_c$ and Φ, in equations (1) and (2), if any, represent reversible conversion of mechanical energy to internal energy and vice versa ? Give reasons to justify your answer.

(f) Which of the terms, $\dot{w}_c$ and Φ, in equations (1) and (2), if any, represent irreversible one-way conversion of mechanical energy to internal energy ? Give reasons to justify your answer.

(g) Which of the terms, $\dot{w}_c$ and Φ, in equations (1) and (2) are associated with generation of entropy ?

[8+4+4+1+1+1+1=20 marks]

Q2. Consider forced convective heat transfer to a Newtonian Fourier fluid flowing in the gap between two parallel horizontal plates separated by a distance L. The lower and upper plates move in their own planes with constant speeds U_1 and U_2. The velocity field is given by $\mathbf{v} = u(y)\,\mathbf{i}$, where y is a vertical coordinate measured from the lower plate and $u(y) = U_1(1 - y/L) + U_2\, y/L$. The lower plate is uniformly heated, with a constant heat flux, q_1, into the fluid. The upper plate is kept at a constant temperature T_2.

(a) Obtain an ordinary differential equation for the temperature distribution $T(y)$. Include the effect of viscous dissipation in your analysis. Express this equation in non-dimensional form, using the variables, $\theta = \dfrac{T - T_2}{(q_1 L/k)}$ and $Y = y/L$, where k is the thermal conductivity of the fluid.

(b) Obtain non-dimensional boundary conditions for solving the differential equation of part (a).

(c) Determine the non-dimensional temperature distribution $\theta(Y)$.

(d) Determine the Nusselt number, $Nu = \dfrac{hL}{k}$, at the upper plate, based on the temperature difference between the two plates, where h is the heat transfer coefficient.

[5+3+6+6=20 marks]

Q3. Consider forced convective heat transfer to a Newtonian Fourier fluid flowing through a circular duct of radius R, surrounded by air at constant ambient temperature T_∞. The axial velocity distribution is given by $w = 2w_{av}(1 - \dfrac{r^2}{R^2})$, where r is the radial coordinate and w_{av} is the average velocity. The temperature distribution is a function of the radial coordinate only, so that the energy equation reduces to $k\dfrac{1}{r}\dfrac{d}{dr}(r\dfrac{dT}{dr}) + \Phi = 0$, where $\Phi = \mu\left(\dfrac{dw}{dr}\right)^2$ is the viscous dissipation, μ and k are respectively the viscosity and thermal conductivity of the fluid. Rotational symmetry requires that $\dfrac{dT}{dr}(0) = 0$ at $r = 0$. Heat transfer from the tube wall to the surrounding air may be represented by a convective boundary condition, with constant effective external heat transfer coefficient h_e. The walls of the duct may be assumed to be perfectly conducting.

(a) Derive a non-dimensional form of the energy equation using the non-dimensional variables, $\theta = (T - T_\infty)/T_\infty$ and $\eta = r/R$.

(b) Write appropriate non-dimensional boundary conditions for solving the differential equation of part (a).

(c) Determine the non-dimensional solution, $\theta(\eta; Br, Bi)$, where $Br = \mu w_{av}^2/(kT_\infty)$ is the Brinkman number and $Bi = h_e(2R)/k$ is the Biot number.

(d) Determine the Nusselt number, $Nu = \dfrac{h(2R)}{k}$, where h is the internal heat transfer coefficient, defined as $h = \dfrac{q_w}{T_w - T_0}$. Here, q_w is the heat flux into the fluid, at the surface, $r = R$, of the duct, T_w is the wall temperature and T_0 is the centre-line temperature.

(e) Determine the limiting form of $\theta(\eta; Br, Bi)$ when $Br \ll 1$ or $Br \to 0$. What physical situation does this limiting case represent ?

(f) Determine the limiting form of $\theta(\eta; Br, Bi)$ when $Bi \gg 1$ or $Bi \to \infty$. What physical situation does this limiting case represent ?

[2+2+7+5+2+2=20marks]

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