Department of Mechanical Engineering
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Class Test Convective Heat and Mass Transfer ME60014/ME61004 Spring 2018

The following information may be useful:

$$\int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}, \qquad \int_0^1 \sin(\lambda_n y)\sin(\lambda_k y)\,dy = \begin{cases} 0, & n \neq k \\ \frac{1}{2}, & n = k, \end{cases} \quad \text{where } \lambda_n = (2n+1)\frac{\pi}{2} \text{ and } n \text{ is an integer.}$$

1. A low Prandtl number constant-property Newtonian-Fourier fluid enters the gap between two large horizontal parallel plates. The speed of the fluid at the inlet is U. The normal distance between the plates is L. The lower plate is maintained at a constant temperature T_w. The upper plate is insulated. The temperature of the fluid at the inlet is T_i. Here, T_w, T_i and U are constants, and $T_w \neq T_i$. For low Prandtl number fluids such as liquid metals ($Pr = \frac{\upsilon}{\alpha} << 1$), the temperature profile in a duct develops more rapidly than the velocity profile. In such a situation ($\upsilon << \alpha$), the temperature profile can be determined based on a uniform velocity profile, $u = U$, where u is the component of velocity in the direction parallel to the plates. This is the inviscid "slug flow" approximation. The effect of viscous dissipation on the temperature distribution may be assumed to be negligible. Introducing the non-dimensional variables, $x = \bar{x}/L$, $y = \bar{y}/L$, $\theta = \frac{T - T_w}{T_i - T_w}$, the energy equation may be expressed as

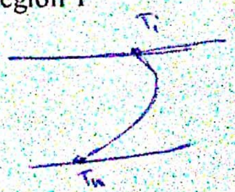
$$\frac{\partial \theta}{\partial x} = \frac{1}{Pe}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right), \tag{1a}$$

where $Pe = UL/\alpha$. Equation (1) has to be solved in the domain, $0 < y < 1$, $0 < x < \infty$, subject to the boundary conditions,

Tw at y=0 Adiabatic Ti at inlet

$$\theta(x,0;Pe) = 0 \text{ at } y = 0, \qquad \frac{\partial \theta}{\partial y}(x,1;Pe) = 0 \text{ at } y = 1, \qquad \theta(0,y;Pe) = 1 \text{ at } x = 0. \tag{1b, c, d}$$

Consider the problem of determining the solution, $\theta(x, y; Pe)$, of the initial boundary-value problem described by equations (1), for large Peclet number flows ($Pe >> 1$).



(a) Obtain an approximate partial differential equation describing the temperature distribution in Region I where $x \sim 1$ and $y \sim 1$, in the limit $Pe \to \infty$.

(b) Determine the solution, $\theta^I(x, y)$, of the differential equation of part (a) satisfying equation (1d).

(c) Does the solution, $\theta^I(x, y)$, satisfy the boundary condition (1c) at $y = 1$?

(d) Does the solution, $\theta^I(x, y)$, satisfy the boundary condition (1b) at $y = 0$?

(e) A thermal boundary layer develops near the lower wall when $Pe >> 1$. In this thermal boundary layer (Region II), $x \sim 1$, $y \sim \delta_0$ where $\delta_0 = \bar{\delta}_0 / L = \frac{1}{Pe^m}$. Use scale analysis to determine the value of m.

(f) Obtain an approximate partial differential equation describing the temperature distribution inside the thermal boundary layer (Region II) in the limit $Pe \to \infty$.

(g) Obtain an approximate solution of the approximate differential equation of part (f) using the integral method discussed in the class. Assume a suitable temperature profile inside the thermal boundary layer. Determine the variation of the thickness, $\delta_T(x)$, of the thermal boundary layer with x.

(h) Use your answer to part (g) to obtain an expression for the variation of the local Nusselt number at the lower plate with x, in Region II.

(i) Is there a thermal boundary layer near the upper wall ? Give reasons to justify your answer.

(j) The temperature distribution, $\theta^{III}(X, y)$, in the downstream region (Region III) where $x \sim Pe$ and $y \sim 1$ may be obtained by solving the equation,

$$\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial y^2}, \tag{2a}$$

in the domain $0 < y < 1$, $0 < X < \infty$, subject to the boundary conditions,

$$\theta(X,0) = 0 \text{ at } y = 0, \quad \frac{\partial \theta}{\partial y}(X,1) = 0 \text{ at } y = 1, \quad \theta(0,y) = 1 \text{ at } X = 0. \tag{2b, c, d}$$

where $X = x / Pe = \bar{x} / (L\,Pe)$. The solution may be expressed as

$$\theta(X,y) = \sum_{n=0}^{\infty} c_n \sin(\lambda_n y) \exp(-\lambda_n^2 X), \tag{3}$$

where $\lambda_n = (2n+1)\dfrac{\pi}{2}$. Determine the coefficients in the series (3).

(k) Show that the bulk mean temperature is given by $\theta_m(X) = \int_0^1 \theta(X,y)\,dy$.

(l) Use the series solution (3) and your answer to part (k) to obtain an expression for $\theta_m(X)$.

(m) Use the series solution (3) and your answer to part (l) to obtain an expression for the variation of the local Nusselt number at the lower plate with X, in Region III.

(n) Determine the numerical value of the Nusselt number at the lower plate in the thermally developed region.

2. The exact solution of the boundary layer energy equation in Region II may be obtained by solving the equation,

$$\frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial Y^2}, \tag{4a}$$

in the domain $0 < Y < \infty$, $0 < x < \infty$, subject to the boundary conditions,

$$\theta(x,0) = 0 \text{ at } Y = 0, \quad \theta \to 1 \text{ as } Y \to \infty, \quad \theta(0,Y) = 1 \text{ at } x = 0. \tag{4b, c, d}$$

where $Y = \sqrt{Pe}\, y$. The solution is

$$\theta = \operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-t^2}\,dt, \tag{5}$$

where $\eta = \dfrac{Y}{2\sqrt{x}}$.

(a) Determine the local Nusselt number at the lower plate in Region II, using the solution (5).

(b) Compare your answers to Q2(a) and Q1(h).