Department of Mechanical Engineering, I.I.T. Kharagpur

ME60014/ME61004 Convective Heat and Mass Transfer Class Test 2 Spring 2018

1. Consider steady laminar two-dimensional incompressible flow of a constant-property Newtonian Fourier gas over a semi-infinite flat plate aligned with the direction of a uniform oncoming isothermal free-stream. The temperature and speed of the approaching stream are T_∞ and u_∞ respectively. **The plate is insulated.** The temperature field is described by the equation,

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{Ec}{\mathrm{Re}}\Phi, \qquad (1)$$

where $\Phi=2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2, x=\frac{\bar{x}}{L}, y=\frac{\bar{y}}{L}, u=\frac{\bar{u}}{\bar{u}_\infty}, v=\frac{\bar{v}}{\bar{u}_\infty}, \theta=\frac{T-T_\infty}{T_\infty}, \mathrm{Re}=\frac{\bar{u}_\infty L}{\nu}$,

$\mathrm{Pr}=\frac{\nu}{\alpha}$, $\mathrm{Ec}=\frac{\bar{u}_\infty^2}{c_pT_\infty}$, L is a reference length, $\bar{x}$ is the coordinate along the plate, $\bar{y}$ is the coordinate normal to the plate, $\bar{u}$ is the component of fluid velocity in the $\bar{x}$-direction, $\bar{v}$ is the component of fluid velocity in the $\bar{y}$-direction, T is the fluid temperature, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity of the fluid, $\alpha=\frac{k}{\rho c_p}$ is the thermal diffusivity, k is the thermal conductivity, ρ is the density, c_p is the specific heat capacity at constant pressure, μ is the viscosity, Re is the Reynolds number, Pr is the Prandtl number and Ec is the Eckert number. The origin of the coordinate system is at the leading edge of the plate.

(a) It is known that when Re is large, $x\sim 1$, $y\sim \mathrm{Re}^{-1/2}$, $u\sim 1$, $v\sim \mathrm{Re}^{-1/2}$, $\theta\sim 1$, inside the boundary layer, except for a small region near the leading edge. Re-write equation (1) using the stretched variables $Y=y\mathrm{Re}^{1/2}$ and $V=v\mathrm{Re}^{1/2}$, appropriate for the inner region or boundary-layer. Derive an approximate equation by taking the inner limit of the energy equation as $\mathrm{Re}\to\infty$ with (x,Y) held fixed.

(b) Write the boundary conditions appropriate for obtaining a solution of the boundary-layer energy equation derived in part (a), in the domain $0\le Y\le\infty$, $0\le x\le\infty$.

(c) The velocity field for high Reynolds number flows may be approximated by $u=F'(\eta)$, $V=\frac{1}{\sqrt{2x}}[\eta F'(\eta)-F(\eta)]$, where $\eta=\frac{Y}{\sqrt{2x}}$ and $F(\eta)$ is the solution of the Blasius equation, $F'''+FF''=0$, with boundary conditions, $F(0)=0, F'(0)=0, F'(\infty)=1$. The mathematical problem described by the boundary-layer energy equation of part (a) and the boundary conditions (b) admits a similarity solution of the form, $\theta(x,Y;\mathrm{Pr},Ec)=Ec^mG(\eta;m,\mathrm{Pr},Ec)$, where m is a constant. Derive an ordinary differential equation for determining the function $G(\eta;m,\mathrm{Pr},Ec)$.

(d) Write boundary conditions appropriate for solving the differential equation derived in part (c).

(e) Determine the value of m for which the function, $G(\eta;m,\mathrm{Pr},Ec)$, is independent of the parameter Ec.

(f) Write the differential equation describing the variation of the function $G(\eta;\mathrm{Pr})$ for the special case when m takes the value obtained in part (e).

(g) Determine the non-dimensional temperature distribution in the limit as the Eckert number approaches zero, that is, $\lim\limits_{Ec\to 0}\theta(x,Y;\mathrm{Pr},Ec)$. **[4+3+5+2+1+1+4 = 20 marks]**