INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Time: 3 Hours

Date: 20-04-2018 AN **Full Marks: 100**

No. of Students: 70 **End Spring Semester Examination 2018**

Sub. No.: ME60014/ME61004 **Sub. Name: Convective Heat and Mass Transfer**

Attempt all questions. This question paper consists of four pages. Symbols have their usual meanings. Please explain your work carefully. Make suitable assumptions wherever necessary. Please state your assumptions clearly. The following information may be useful:

$$I_m = \int_0^1 6\eta(1-\eta)\eta^m \, d\eta = \frac{6}{(m+2)(m+3)}, \text{ when } m > -2. \quad J_m = \int_0^1 2\eta\, \eta^m \, d\eta = \frac{2}{(m+2)} \text{ when } m > -2.$$

1. Consider steady laminar forced convective heat transfer to a constant-property Newtonian Fourier fluid flowing through the gap between two large horizontal parallel plates, driven by a constant applied pressure gradient in the x-direction. The upper plate is uniformly heated, with a constant heat flux, q_0, into the fluid, for $x > 0$. Heat transfer from the upper plate to the fluid is negligible for $x < 0$. The lower plate is insulated. The temperature of the fluid at $x = 0$ is T_{in}. The fully developed velocity distribution is given by $u = 6u_{av}\eta(1-\eta)$, where u is the component of velocity in the x-direction, u_{av} is the average velocity through the channel, $\eta = \frac{y}{L}$, L is the normal distance between the plates and y is the coordinate normal to the plates, measured from the lower plate. The plates have infinite span (in the z-direction). The effect of viscous dissipation and axial conduction of heat in the fluid are negligible.

(a) By integrating the energy equation across the channel, or otherwise, obtain an expression for the bulk temperature gradient, $\frac{dT_m}{dx}$, for $x > 0$, where T_m is the bulk mean temperature of the fluid.

(b) In the thermally developed region of the flow, $T - T_m = \psi_1(\eta)$, where T is the fluid temperature. Determine the function $\psi_1(\eta)$.

(c) Obtain a partial differential equation for the function $\psi = T - T_m - \psi_1(\eta)$. Determine the temperature distribution in the thermal entrance region by solving this partial differential equation, subject to appropriate boundary and initial conditions, using the method of separation of variables. Obtain expressions for the coefficients in the infinite series solution using the orthogonality property of the eigenfunctions.

(d) Using your answer to parts (b) and (c), obtain an expression for the local Nusselt number at the upper plate.

(e) Use your answer to part (d) to obtain the numerical value of the Nusselt number at the upper plate in the thermally developed region of the flow.

[2+4+8+2+2=18 marks]

2. Solid food is heated while flowing through a circular tube of radius R at constant speed U. The heat flux at the surface of the tube is maintained at a constant value q_0. The effect of axial conduction of heat may be neglected when the Peclet number is large.

(a) Obtain an expression for the bulk temperature gradient, $\frac{dT_m}{dz}$, where T_m is the bulk mean temperature of the moving solid and z is the axial coordinate.

(b) In the thermally developed region of the flow, $T - T_m = \psi(\eta)$, where $\eta = \frac{r}{R}$ and r is the radial coordinate. Determine the function $\psi(\eta)$.

(c) Determine the numerical value of the Nusselt number based on the duct diameter, in the thermally developed region.

[4+4+4=12 marks]

3. Consider steady laminar two-dimensional forced convection boundary-layer flow over a heated semi-infinite flat plate maintained at a constant temperature, T_w, higher than the ambient temperature T_∞. The plate is aligned with the direction of a uniform isothermal oncoming free-stream. Assume that $\bar{\delta}_0 / L << 1$, $\bar{\delta}_{T_0} / L << 1$, and $\bar{\delta}_{T_0} / \bar{\delta}_0 >> 1$. Here, $\bar{\delta}_0$ and $\bar{\delta}_{T_0}$ are respectively the orders of magnitude of the hydrodynamic and thermal boundary layers, and L is the distance from the leading edge to a reference point on the plate. The local Nusselt number is given by $Nu_x = c(Pr)\sqrt{Re_x}$, where Re_x is the local Reynolds number and Pr is the Prandtl number. When $\bar{\delta}_{T_0} / \bar{\delta}_0 >> 1$, the velocity distribution in the boundary layer may be approximated by $\bar{u} = \bar{u}_\infty$, $\bar{v} = 0$, where $\bar{u}$ and $\bar{v}$ are respectively the components of the fluid velocity in the $\bar{x}$ and $\bar{y}$ directions, $\bar{x}$ is the coordinate along the plate, $\bar{y}$ is the coordinate normal to the plate and $\bar{u}_\infty$ is the free-stream speed.

(a) Use scale analysis to show that $\bar{\delta}_{T_0} / \bar{\delta}_0 >> 1$ when $Pr << 1$.

(b) It is known that $c(Pr) \rightarrow K_1 Pr^m$ as $Pr \rightarrow 0$, where K_1 and m are constants. Use scale analysis to obtain an estimate for the value of m.

(c) Obtain an approximate solution of the boundary-layer energy equation using the integral method when $Pr << 1$. Assume that the temperature distribution is given by $\theta = \dfrac{T - T_w}{T_\infty - T_w} = \sin\left(\dfrac{\pi}{2}\eta\right)$ where $\eta = \dfrac{\bar{y}}{\bar{\delta}_T(\bar{x})}$ and $\bar{\delta}_T(\bar{x})$ is the thickness of the heated layer. Determine $\bar{\delta}_T(\bar{x})$.

(d) Use your answer to part (c) to obtain an estimate for the value of K_1. **[3+3+8+6=20 marks]**

4. *Consider natural-convection* boundary-layer flow of a Newtonian Fourier fluid along a semi-infinite *vertical flat plate kept at a constant temperature,* T_w, higher than the ambient temperature, T_∞. The variation *of the density of the fluid* with temperature, T, may be approximated by the linearized equation of state, $\rho(T) = \rho_\infty[1 - \beta_\infty(T - T_\infty)]$, where ρ_∞ and β_∞ are respectively the density and coefficient of thermal expansion of the fluid at temperature T_∞. The flow is described by the equations, $\dfrac{\partial \bar{u}}{\partial \bar{x}} + \dfrac{\partial \bar{v}}{\partial \bar{y}} = 0$,

$$\rho_\infty\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = f_{buoy} + \mu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \quad \rho_\infty c_p\left(\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) = k\frac{\partial^2 T}{\partial \bar{y}^2},$$

where f_{buoy} is the buoyancy force per unit volume, defined by $f_{buoy} = -\dfrac{d\bar{p}}{d\bar{x}} - \rho(T)g$. Here, $\bar{x}$ and $\bar{y}$ are respectively Cartesian coordinates along and normal to the plate, $\bar{u}$ and $\bar{v}$ are respectively the components of the fluid velocity in the $\bar{x}$ and $\bar{y}$ directions, g is the gravitational acceleration, μ, c_p and k are respectively the viscosity, specific heat capacity at constant pressure and thermal conductivity of the fluid at the reference state and $\bar{p}$ is the pressure. The origin of the coordinate system is at the leading edge of the plate; the positive $\bar{x}$-direction is upwards.

(a) Show that $f_{buoy} = (\rho_\infty - \rho)g$.

(b) Using the given equation of state, obtain an expression for f_{buoy} in terms of ρ_∞, β_∞, g, T and T_∞.

(c) Using your answer to part (b), non-dimensionalize the governing equations by introducing the following non-dimensional variables: $x = \dfrac{\bar{x}}{L}$, $y = \dfrac{\bar{y}}{L}$, $u = \dfrac{\bar{u}}{\bar{u}_0}$, $v = \dfrac{\bar{v}}{\bar{u}_0}$, $\theta = \dfrac{T - T_\infty}{\Delta T}$, where L is a reference length, $\bar{u}_0$ is a reference speed and $\Delta T = T_w - T_\infty$.

$\Delta T \ \theta = T$

(d) Use scale analysis to obtain an estimate for the order of magnitude, $\bar{u}_0$, of the vertical component of the velocity at distance L from the leading edge of the plate.

(e) Use scale analysis to obtain estimates for the ratios, $v_0 = \frac{\bar{v}_0}{\bar{u}_0}$ and $\delta_0 = \frac{\bar{\delta}_0}{L}$, where $\bar{v}_0$ is the order of magnitude of the horizontal component of the fluid velocity at distance L from the leading edge of the plate and $\bar{\delta}_0$ is the order of the thickness of the boundary layer.

(f) Write appropriate boundary conditions for solving the system of equations of part (c).

(g) The equations of part (c) admit a self-similar solution of the form, $\psi = x^a F(\eta)$, $\theta = x^b G(\eta)$, satisfying the boundary conditions of part (f), where ψ is the streamfunction defined by the equations $u = \frac{\partial \psi}{\partial Y}$, $V = -\frac{\partial \psi}{\partial x}$, $Y = \frac{y}{\delta_0}$, $V = \frac{v}{v_0}$ and $\eta = \frac{Y}{x^m}$, for some values of a, b and m. Determine these values of a, b, and m.

(h) Using your answer to part (g), determine the ratio, $\frac{q_{w2}}{q_{w1}}$, where q_{w1} and q_{w2} are respectively the wall heat fluxes at distances $\bar{x}_1 = L$ and $\bar{x}_2 = 16L$ from the leading edge of the plate.

[1+1+2+1+2+2+3+3= 15 marks]

5. Consider the thermal entrance problem in a long heated isothermal circular tube of radius R. The flow is steady laminar axisymmetric hydrodynamically developed and incompressible. The axial velocity through the duct is given by $\bar{w} = 2\bar{w}_{av}\left[1-r^2\right]$, where $r = \bar{r}/R$, $\bar{r}$ is the radial coordinate and $\bar{w}_{av}$ is the average velocity through the duct. The temperature of the tube wall is maintained at a constant value, T_w, for $\bar{z} \geq 0$, where $\bar{z}$ is the axial coordinate oriented along the axis of the tube in the direction of the flow. Heat transfer from the pipe wall upstream of $\bar{z} = 0$ to the surroundings is negligible. The temperature of the fluid at the inlet of the duct is T_{in}, where T_{in} is a constant less than T_w. The effect of viscous dissipation on the temperature distribution is negligible. When the Peclet number is large, the non-dimensional temperature, $\theta = \frac{T_w - T}{T_w - T_{in}}$, downstream of $\bar{z} = 0$ may be expressed as $\theta(r,Z) = \sum_{n=0}^{\infty} c_n f_n(r)\exp\left(-\beta_n^2 Z\right)$, where $f_n(r)$ is the nth Graetz function, obtained by solving the equation, $\frac{d}{dr}(r\frac{df_n}{dr}) + \beta_n^2\, r\left(1-r^2\right) f_n = 0$, with boundary condition, $f_n(1) = 0$, satisfying the symmetry condition, $f_n'(0) = 0$, at the axis of the pipe and the normalization, $f_n(0) = 1$. Here, $Z = \bar{z}/(RPe)$, $Pe = \bar{w}_{av}(2R)/\alpha$ is the Peclet number and α is the thermal diffusivity of the fluid. The eigenvalues, $\mu_n = \beta_n^2$, of the Graetz problem are ordered so that $\mu_0 < \mu_1 < \mu_2 < \ldots$. The leading eigenvalues are given in the following table:

n	0	1	2	3	4
$\mu_n = \beta_n^2$	7.3135868	44.609468	113.92104	215.24058	348.564045

(a) Show that the Graetz functions satisfy the relation, $\int_0^1 r\left(1-r^2\right) f_n(r)\,dr = -\frac{f_n'(1)}{\mu_n}$.

(b) Using the series solution and the result of part (a), show that the non-dimensional bulk-temperature may be expressed as $\theta_m(Z) = -4\sum_{n=0}^{\infty}\frac{c_n f_n'(1)}{\mu_n}e^{-\mu_n Z}$ for $Z > 0$.

(c) Obtain an expression for the local Nusselt number, $\mathrm{Nu}(Z)$ for $Z > 0$. Express your answer in terms of c_n, μ_n and other appropriate quantities.

(d) Using the given table, determine the numerical value of the Nusselt number in the thermally developed region.

(e) Show that $\frac{d\varphi}{dZ} = -2Nu(Z)\varphi$ for $Z > 0$, where $\varphi = T_w - T_m$ and T_m is the bulk mean temperature of the fluid.

(f) Obtain the solution of the differential equation of part (e) in the thermally developed region.

(g) Using your answer to part (f), obtain an expression for the heat flux, $q_w(Z)$, into the fluid at the tube wall, in the thermally developed region.

(h) The wall heat flux at an axial location, $Z = Z_0$, in the thermally developed region is $q_w(Z_0) = q_0$. Using your answer to part (g), obtain an expression for the rate of heat transfer into the fluid through the tube wall for $Z > Z_0$. Express your answer in terms of q_0.

[3+3+3+1+7+2+2+4=25 marks]

6. Give *concise* answers to the following parts, with proper justification.

(a) Consider steady laminar incompressible axisymmetric hydrodynamically and thermally developed forced convective heat transfer to a constant-property Newtonian Fourier fluid in a circular tube with uniform surface heat flux for the case when viscous dissipation is negligible. The solution for the temperature field presented in most textbooks is obtained by assuming that the effect of axial conduction of heat in the fluid is negligible. A student claims that neglecting the axial conduction term in the energy equation does not introduce any error in the solution for the temperature field, even if the Peclet number of the flow is small. Do you agree with the student ?

(b) Consider steady laminar forced-convection boundary-layer flow of a constant-property Newtonian Fourier fluid along a semi-infinite heated flat plate aligned with the direction of a uniform isothermal oncoming free-stream. The plate is subjected to a constant surface heat flux. A student argues that if the Prandtl number of the fluid is Pr = 1, the temperature field can be predicted from a knowledge of the velocity field, without solving the energy equation, using Reynolds analogy. Do you agree with the student ?

(c) An engineer has obtained an estimate of the rate of heat transfer by natural convection from a long heated isothermal vertical cylinder, using the correlations for the local Nusselt number for natural convection along a large heated isothermal vertical flat plate. The engineer thinks that his estimate is correct when the Grashof number is large. Do you agree with the engineer ?

(d) Consider forced convective heat transfer from a large heated isothermal plate aligned with the direction of a uniform isothermal oncoming free-stream. An engineer wants to double the rate of heat transfer from the plate by increasing the free-stream speed. By what factor must the free-stream speed be increased to achieve this ?

(e) Consider steady laminar axisymmetric hydrodynamically developed and thermally developing forced convection in a circular tube with step change in the wall temperature. A student argues that the thermal entrance length is larger in the case of forced convection of oil than in the case of forced convection of water when the average velocity though the duct is the same for the two cases. Do you agree with the student ?

[2+2+2+2+2=10 marks]

END OF PAPER

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