# Indian Institute of Technology, Kharagpur

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| Date of Examination: 24 Nov 2017 | Time: 3Hrs. Full Marks 100 |
| End Semester Exam. 2015 | **3th Yr (MTech+Dual Degree) Thermal** |
| Sub No. ME60017 | Sub Name: **Conduction and Radiation** |
| No. of Students: 42 | of the dept. of: **Mech Engg** |

**Answer all questions.** Marks are given on the left margin in box
Wherever necessary, make suitable assumptions and state them clearly

1. Consider a one-dimensional transient heat conduction in a slab of thickness $L$. The slab is initially at constant temperature, $T_0$, at time $t = 0$. The temperature of the face, $x = 0$, of the slab is suddenly increased to a constant value, $T_1$, at time $t = 0$, and maintained at temperature $T_1$ for times $t > 0$, where $T_1 > T_0$. The temperature of the face, $x = L$, is kept at the constant value, $T_0$, for times $t > 0$. The thermal diffusivity, $\alpha$, of the slab may be assumed constant.

$\boxed{2}$ (a) Determine the steady state temperature distribution, $T_{ss}(x)$, in the slab.

$\boxed{4}$ (b) Formulate the initial boundary value problem for determining the temperature field, $T(x,t)$, in terms of the variable $u(x,t) = T(x,t) - T_{ss}(x)$. Obtain a partial differential equation for the function $u(x,t)$ and write the appropriate boundary and initial conditions for determining the solution, $u(x,t)$, of this equation in the domain $0 < x < L$ for $0 < t < \infty$.

$\boxed{2}$ (c) Show that

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = -I_2$$

$$\text{where} \quad I_1(t) = \int_0^L \frac{1}{2}[u(x,t)]^2\,\mathrm{d}x \quad \text{is the 'solution energy'}$$

$$\text{and } {}^{*}I_2(t) = \alpha \int_0^L \frac{1}{2}\left[\frac{\partial u}{\partial x}(x,t)\right]^2 \mathrm{d}x \quad \text{is the 'thermal dissipation'}$$

$\boxed{2}$ (d) Use your answer to part(c) to show that $\lim_{t\to\infty} u(x,t) = 0$ for arbitrary initial conditions.

$\boxed{12}$ (e) Use the method of *separation of variables* to determine the solution, $u(x,t)$, of the initial boundary value problem of part(b).

$\boxed{2}$ (f) An Engineer thinks that steady state will be reached at time $t_f = K\frac{L^2}{\alpha}$, where $K$ is a dimensionless constant. Do you agree with the engineer? Give reasons to justify your answer.

$\boxed{12}$ 2. On an overcast day the directional behaviour of the intensity of solar radiation reaching the surface of the earth after being scattered by the atmosphere may be approximated as $I_{sky}(\theta) = I_{sky}(0) \times \cos\theta$, where $\theta$ is measured from the surface normal. For a day with $I_{sky}(0) = 100\,\mathrm{W/m^2.sr}$ determine the solar radiation in $\mathrm{W/m^2}$ hitting a solar collector that is horizontal. Neglect any radiation from the earths surface hitting the solar collector.

$\boxed{12}$ 3. It is possible to manufacture a diffusely absorbing/emitting selective absorber with a spectral emmisivity $\epsilon_\lambda = \epsilon_s = 0.05$ for $0 < \lambda < \lambda_c$ and $\epsilon_\lambda = \epsilon_c = 0.95$ for $\lambda > \lambda_c$, where $\lambda_c$ is the cut-off wavelength. A solar collector has the above selective absorber plate with cut-off wavelength $\lambda_c = 3.0\mu m$ that is at a temperature of 350 $K$ and is exposed to solar radiation of $q_{sun} = 1000\,\mathrm{W/m^2}$ at an angle of $\theta_s = 30°$ off-normal. Determine the net radiative energy gain $q_{net}$ in $\mathrm{W/m^2}$ for such a solar collector. The temperaure of the sun is 5800 K. The sky is assumed to be very cold.

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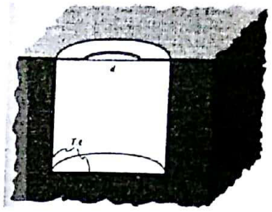
4. A cylindrical cavity in a graphite block has a hole of diameter $D$ and length $L$. The top of the cavity is covered with a disk, which has a hole of diameter $d$. The entire inside of the cavity is isothermal at temperature $T$, and grey, diffuse with emissivity $\epsilon$. The entire cavity surface can be considered as a single zone.



$\boxed{8}$ (a) Determine the amount of radiation escaping from the cavity through the small hole.

$\boxed{4}$ (b) What is the effective emissivity of the hole of diameter $d$ if $d \ll D$ and $d \ll L$.

$\boxed{12}$ 5. Two very large parallel isothermal plates of emissivity $\epsilon = 0.5$ and $\epsilon = 0.8$ exchange heat by radiation. Find the percentage reduction in radiation heat transfer when a polished aluminum plate of emissivity $\epsilon = 0.05$ is placed between them. Show the radiation network diagram. Assume diffuse gray surfaces.

6. Starting from the equation of radiative heat transfer show that

$\boxed{8}$ (a) The divergence of spectral radiation heat flux is

$$\nabla \cdot \mathbf{q}_\eta = \kappa_\eta (4\pi I_{b\eta} - G_\eta)$$

where, $G$ is the *incident radiation function* and $\kappa$ is the *absorption coefficient*

$\boxed{4}$ (b) The divergence of radiation heat flux for a grey medium is

$$\nabla \cdot \mathbf{q} = \kappa (4\sigma T^4 - G)$$

7. A 1mm thick slab of an absorbing/emitting gas has approximately linear temperature distribution between $T_1 = 1000\text{K}$ and $T_2 = 2000\text{K}$. On both sides, the medium is bounded by vacuum with non-reflecting boundaries.

$\boxed{4}$ (a) Write down the equation of radiative transport for the absorbing/emitting gray medium with no scattering.

$\boxed{6}$ (b) If the medium has a constant and gray absorption coefficient of $\kappa = 1m^{-1}$, what is the radiative intensity (as a function of direction) leaving the hot side of the slab?

$\boxed{6}$ (c) Give an expression for radiative heat flux leaving the hot side.

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## The fractional blackbody emissive power

The Stefan-Boltzmann Constant $\sigma = 5.67 \times 10^{-8} \mathrm{W/m^2.K^4}$

| $n\lambda T$ [$\mu$mK] | $\eta/nT$ [cm$^{-1}$/K] | $E_{b\lambda}/n^3T^5$ [W/m$^2\mu$mK$^5$] | $E_{b\eta}/nT^3$ [W/m$^2$cm$^{-1}$K$^3$] | $f(n\lambda T)$ |
|---|---|---|---|---|
| 1000 | 10.0000 | $0.02110 \times 10^{-11}$ | $0.00211 \times 10^{-8}$ | 0.00032 |
| 1100 | 9.0909 | 0.04846 | 0.00586 | 0.00091 |
| 1200 | 8.3333 | 0.09329 | 0.01343 | 0.00213 |
| 1300 | 7.6923 | 0.15724 | 0.02657 | 0.00432 |
| 1400 | 7.1429 | 0.23932 | 0.04691 | 0.00779 |
| 1500 | 6.6667 | 0.33631 | 0.07567 | 0.01285 |
| 1600 | 6.2500 | 0.44359 | 0.11356 | 0.01972 |
| 1700 | 5.8824 | 0.55603 | 0.16069 | 0.02853 |
| 1800 | 5.5556 | 0.66872 | 0.21666 | 0.03934 |
| 1900 | 5.2632 | 0.77736 | 0.28063 | 0.05210 |
| 2000 | 5.0000 | 0.87858 | 0.35143 | 0.06672 |
| 2100 | 4.7619 | 0.96994 | 0.42774 | 0.08305 |
| 2200 | 4.5455 | 1.04990 | 0.50815 | 0.10088 |
| 2300 | 4.3478 | 1.11768 | 0.59125 | 0.12002 |
| 2400 | 4.1667 | 1.17314 | 0.67573 | 0.14025 |
| 2500 | 4.0000 | 1.21659 | 0.76037 | 0.16135 |
| 2600 | 3.8462 | 1.24868 | 0.84411 | 0.18311 |
| 2700 | 3.7037 | 1.27029 | 0.92604 | 0.20535 |
| 2800 | 3.5714 | 1.28242 | 1.00542 | 0.22788 |
| 2900 | 3.4483 | 1.28612 | 1.08162 | 0.25055 |
| 14,200 | 0.7042 | 0.03693 | 0.74456 | 0.96418 |
| 14,400 | 0.6944 | 0.03520 | 0.72998 | 0.96546 |
| 14,600 | 0.6849 | 0.03358 | 0.71579 | 0.96667 |
| 14,800 | 0.6757 | 0.03205 | 0.70198 | 0.96783 |
| 15,000 | 0.6667 | 0.03060 | 0.68853 | 0.96893 |
| 15,200 | 0.6579 | 0.02923 | 0.67544 | 0.96999 |
| 15,400 | 0.6494 | 0.02794 | 0.66270 | 0.97100 |
| 15,600 | 0.6410 | 0.02672 | 0.65029 | 0.97196 |
| 15,800 | 0.6329 | 0.02556 | 0.63820 | 0.97288 |
| 16,000 | 0.6250 | 0.02447 | 0.62643 | 0.97377 |
| 16,200 | 0.6173 | 0.02343 | 0.61496 | 0.97461 |
| 16,400 | 0.6098 | 0.02245 | 0.60379 | 0.97542 |
| 16,600 | 0.6024 | 0.02152 | 0.59290 | 0.97620 |
| 16,800 | 0.5952 | 0.02063 | 0.58228 | 0.97694 |
| 17,000 | 0.5882 | 0.01979 | 0.57194 | 0.97765 |
| 17,200 | 0.5814 | 0.01899 | 0.56186 | 0.97834 |
| 17,400 | 0.5747 | 0.01823 | 0.55202 | 0.97899 |
| 17,600 | 0.5682 | 0.01751 | 0.54243 | 0.97962 |
| 17,800 | 0.5618 | 0.01682 | 0.53308 | 0.98023 |
| 18,000 | 0.5556 | 0.01617 | 0.52396 | 0.98081 |
| 18,200 | 0.5495 | 0.01555 | 0.51506 | 0.98137 |
| 18,400 | 0.5435 | 0.01496 | 0.50638 | 0.98191 |
| 18,600 | 0.5376 | 0.01439 | 0.49790 | 0.98243 |
| 18,800 | 0.5319 | 0.01385 | 0.48963 | 0.98293 |

End of exam