INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: 26-09-2018 AN **Time: 2 Hours** **Full Marks: 120**
No. of Students: 22 **Mid Spring Semester Examination 2018**
Sub. No.: ME60017 **Sub. Name: Conduction and Radiation Heat Transfer**

Instructions: Attempt all questions. Symbols have their usual meanings. Please explain your work carefully. **Clearly indicate the coordinate system used in your analysis.** Make suitable assumptions wherever necessary. Please state your assumptions clearly. The following information may be useful:
The Laplacian operator in cylindrical coordinates is

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}.$$

1. Consider conduction of heat in a solid. The component of the heat flux vector, $\mathbf{q}$, along a given direction ox depends on a linear combination of the temperature gradients in the ox, oy and oz directions when the solid is anisotropic. The constitutive relation for multidimensional transient heat conduction in an anisotropic solid may be expressed as

$$-q_x = K_{11}\frac{\partial T}{\partial x} + K_{12}\frac{\partial T}{\partial y} + K_{13}\frac{\partial T}{\partial z}, \tag{1a}$$

$$-q_y = K_{21}\frac{\partial T}{\partial x} + K_{22}\frac{\partial T}{\partial y} + K_{23}\frac{\partial T}{\partial z}, \tag{1b}$$

$$-q_z = K_{31}\frac{\partial T}{\partial x} + K_{32}\frac{\partial T}{\partial y} + K_{33}\frac{\partial T}{\partial z}, \tag{1c}$$

where $T(x,y,z,t)$ represents the temperature field, x, y and z are the Cartesian coordinates of a point, o is the origin of the coordinate system, t is the time, q_x, q_y and q_z are the Cartesian components of the heat flux vector and K_{ij} are the elements of the thermal conductivity tensor.

(a) Starting with the general constitutive relation given by equations (1), use appropriate coordinate transformations to deduce the form of the thermal conductivity tensor for an isotropic solid.

(b) The constitutive relation for steady one-dimensional heat flow in the x-direction in an isotropic solid is Fourier's law of heat conduction which states that

$$q_x = -k(T)\frac{dT}{dx}(x), \tag{2}$$

where $k(T)$ is the thermal conductivity of the solid. Using equations (1) and (2) and your answer to part (a), obtain expressions for the elements of the thermal conductivity tensor for an isotropic solid in terms of the thermal conductivity $k(T)$.

(c) Using your answer to part (b), express Fourier's law of heat conduction in vector form, for the case of multidimensional transient heat conduction in an isotropic solid. **[15+3+2=20 marks]**

2. Consider steady two-dimensional heat conduction in a long solid circular cylinder of radius R. The temperature at the surface, $r = R$, of the cylinder varies with the azimuthal angle as $T_S = f(\phi)$, where $f(\phi) = T_0 + T_1\cos(\phi) + T_2\sin(2\phi)$, and T_0, T_1 and T_2 are constants. Here, ϕ is the azimuthal angle in a cylindrical coordinate system with origin at the axis of the cylinder and r is the radial coordinate. Determine the steady state temperature distribution, $T(r,\phi)$, in the solid cylinder. **[30 marks]**

3. A semi-infinite solid is initially at constant temperature T_0.

(a) Determine the temperature distribution in the semi-infinite solid as $t \to \infty$ when the temperature, $T_S(t)$, of the face, $x = 0$, of the slab varies periodically with time as $T_S(t) = T_0 + T_1 \sin(\omega t)$, for times $t > 0$. Here, T_1 and ω are constants.

(b) Using your answer to part (a), determine the steady periodic temperature distribution in the semi-infinite solid as $t \to \infty$ when the surface temperature varies with time as $T_S(t) = T_0 + T_1 \sin(\omega t) + T_2 \sin(2\omega t)$, where T_2 is a constant.

[20+10=30 marks]

4. Consider one-dimensional transient heat conduction in a solid slab of thickness L, constant thermal conductivity k, constant density ρ and constant specific heat capacity c. The rate of volumetric heat generation, q''', in the solid is constant. The initial temperature of the slab is T_i at time $t = 0$, where T_i is a constant. The heat flux, q_x, at the face, $x = 0$, of the slab is maintained at a constant value, q''_0, for times $t > 0$. The face, $x = L$, of the slab is insulated.

(a) Obtain an ordinary differential equation describing the variation of the mean temperature, $T_m(t) = \frac{1}{L}\int_0^L T(x,t)\,dx$, of the slab with time, where $T(x,t)$ is the temperature field.

(b) Solve the differential equation of part (a) with appropriate initial condition, to determine $T_m(t)$.

(c) Formulate the initial boundary value problem for determining the temperature field in terms of the variable $u(x,t) = T(x,t) - T_m(t)$. Obtain a partial differential equation for the function $u(x,t)$ and write appropriate boundary and initial conditions for determining the solution, $u(x,t)$, of this equation in the domain $0 < x < L$ for $0 < t < \infty$.

(d) Determine the solution of the initial boundary value problem of part (c).

(e) An engineer thinks that the temperature field, $T(x,t)$, will become steady as $t \to \infty$ if the heat flux, q''_0, at the surface, $x = 0$, of the slab and the rate of volumetric heat generation, q''', satisfy a certain relation. Do you agree with the engineer ? If you agree, determine the relation between q''_0 and q''' for the temperature field to be independent of time as $t \to \infty$. If you disagree, give reasons to justify your answer.

[6+3+6+20+5=40 marks]

END OF PAPER