5/1/17

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Books: Shingley's Mechanical engineering design
Budynas and Nisbett

Monday – Tutorial

Design: Formulating a solution/ plan to a specified task

Eg: Bicycle : To reach a place faster and with lesser effort than walking

Requirement : Product must be functionally safe, reliable, usable, manufacturable, competitive, marketable

Note : There is no unique solution
use high performance alloy for critical load bearing parts of bicycle.
⇒ light weight but expensive
⇒ Need to optimise based on constraints.
At times a few criteria are given more important than others.

steps ① understand the problem/need
② Identify the known quantities
③ Identify the unknown and find a strategy for solving
④ state all assumptions and decisions
⑤ Analyze the problem.
⑥ Evaluate your solution / modify if needed
⑦ Present it.

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* Man is a tool making animal

* Design: Innovative and highly iterative process.

MPI → Multi Point Injection Engine

IC engine efficiency ~ 40%

Powerplant efficiency ~ 36%

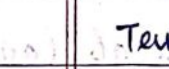
Electricity Distribution efficiency ~ 80%

∴ Electric cars may not be that efficient

Piston Wear — Cylinder lining

Springs

Tension



Compression

Leaf spring in trucks

* More load carrying capacity for same size
* Laterally stable

11 Jan

Need a mechanism to open and close valves for air intake and exhaust for an IC engine.

Known : • Opening and closing time for the valves and duration.

⇒ interpret in terms of crank rotation.

Synchronize valves with cam-shaft rotation.

timing chain / timing belt

- Temperature range of operation
- Maximum pressure on side engine.

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Questions :- Overall mechanism
- Material &

Example :- Cam (wear should be minimum)
- Components
- Power supply

Components - crank-shaft
- camshaft + timing chain.
- Rocker arm] Via Ball and Socket Joint
- Push Rod]
- Valve Assembly (Valve + Spring)

Petrol : Air + Fuel Mixture Diesel : Fuel Injector

Keep in Mind - Cost
- Standard Materials, Parts
- Use codes for analysis, design manufacturing.
- stress and strength
  - stress: Depends on applied external load
  - strength: Material property
- uncertainity
  - → due to manufacturing process
  - → environment
  - → composition of material
  - → Loading
  - → Validity of mathematical model and approximation

To account for uncertainity :-

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Design factor = minimum breaking strength / permissible working load

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↳ Use a design factor (n_d)

breking/failure of an element (weakest)

n_d = loss of function parameter / Maximum Allowable value of Parameter

= loss of function strength / Allowable stress. (for strength based design)

= loss of function load / Max. Allowable load.

Typically $n_d > 1$

Example: For a structure

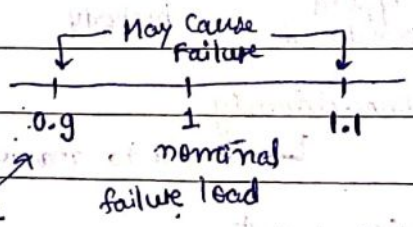
stress ← Maximum load is known with uncertainity ± 5%

Strength ← Load causing failure has uncertainity = ± 10%

Calculate design factor

Loss of function:

Failure load has uncertainity = ± 10%



Design assuming that failure load is 0.9 × nominal failure load

Nominal failure load = $\frac{1}{0.9}$ × design

⇒ increase the loss of function parameter value

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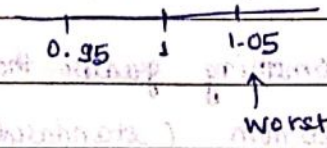
Factor of Safety = $\frac{\text{Load that breaks the product}}{\text{Normal load on the product}}$

$n_d = \frac{1}{0.9}$ (Assuming no uncertainity in maximum load)

Maximum allowable load : uncertainity ± 5%



∴ Design load = 1.05 × nominal load.

$n_d = \frac{1}{1/1.05} = 1.05$

Design factor with both uncertainities

$n_d = \frac{1/0.9}{1/1.05} = 1.167$

12th Jan

Factor of safety:

After design is completed with a specified design factor.

- standard sizes are employed.
- standard components are of higher ratings etc.

⇒ Final (actual) design will be somewhat different from the initial one.

Factor of safety, $n_f = n_d$ after finalizing standard design and component.

Example

A solid circular rod is under torsion

Torque T = 100 Nm

Material strength = 85 MPa (shear)

Design factor = 2.5

Maximum allowable stress = $\frac{85}{2.5}$ MPa ← n_d

$\tau = \frac{Tr}{J} = \frac{T \cdot d/2}{\pi d^4} \times 32$

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$\Rightarrow \tau = \frac{16T}{\pi d^3}$ (Max. stress at surface)

$\therefore \frac{16T}{\pi d^3} = \frac{85}{2.5} \Rightarrow d = 24.66$ mm.
↳ difficult to manuf.

Something greater than 24.66 mm is needed

$d = 25$ mm (standard size).

$\therefore$ Actual stress $= \frac{16T}{\pi d^3} = 32.59$ MPa (with 25 mm)

Engineering Materials

Ferrous alloys — predominantly used.

① Steel — (alloy of iron and carbon)

Other elements — Cr, Ni, Mn, Si, Mb, V
(in small percentage)

High hardness/toughness ⇒ low ductility

② Alloy steel (other elements are present in significant percentage)

- Cr steel ← increases hardness, maintains ductility
- Ni steel ← Increases toughness
- W Steel

③ Cast iron.

Non Ferrous

① Al

② Titanium

③ Cu Based alloys.

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Non Metal

Plastic, Wood, Ceramic, Composit

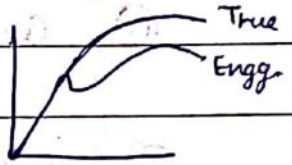
Carbon Nanotube – Tera Pascal GPa

Fe single crystal — less than steel

18th Jan. Modes of Mechanical failure

① Force or temperature induced large elastic deformation,

② Yielding – Ductile – Accumulated plastic deformation in ductile material.



③ Ductile Rupture – breaking of a ductile specimen when ultimate strength is reached.

④ Brinelling – Localized yielding induced by static forces between two curved surfaces in contact.
(common in bearings)

⑤ Brittle ~~fracture~~ fracture: failure in brittle materials when loaded beyond elastic limit.

tension – 90° Torsion – 45° (due to max normal stress)

⑥ FATIGUE:

Cyclic loading: Even below Yield point

(No warning) – Sudden and catastrophic failure due to fluctuating load/deformation.

→ High Cycle Fatigue > 50,000 cycles

→ Low Cycle Fatigue < 10,000 cycles.

→ 10^6 cycles + → ∞ cycle fatigue

→ Thermal fatigue: due to temperature fluctuation

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(7) Surface fatigue – in rolling surfaces in contact.
– pitting, spalling.

(8) Corrosion and wear

(9) Impact

(10) ~~Fretting~~ Fretting – between two solid surfaces under normal force – when small amplitude cycle of relative motion is applied.

(11) Creep – Accumulation of plastic deformation over long time.

(12) Galling – Massive surface destruction between two sliding surfaces.

(13) Spalling – a small particle getting dislodged spontaneously from surface

(14) Buckling – ~~Excessive deformation~~ sudden increase in deflection under compressive loading.

Material selection

Steps (1) Analyze the material specific requirements of the applications.

Need to think of (1) operational ~~aspects~~ aspects / functional aspects
(2) Potential failure modes.
(3) Market related factors.

Potential Application Ratio.

Strength to volume ratio – Corresponding material property
Strength to weight ratio – Yield strength or ~~Ultimate~~ ultimate strength (Ultimate/Yield strength) / density

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Strength at high temperature — Strength variation with temperature

Dimensional stability at high temperature — Thermal expansion coeffiient

" " under temp. fluctuation — "

Stiffness — Elastic Modulus

Ductility — % elongation

Ability to store energy elastically — Resilience

" " " " plastically — Toughness

Wear resistance —

Corrosion resistance —

Cost constraint —

Procurement time constraint —

Step (2) (a) Give weightage to requirements

(b) Identify a few superior materials for each requirements

Step (3) Identify the common materials from the lists.

Choose the best depending on weightage.

* Define a performance metric P depending on application.

For a structural element P depends on

— functional requirements

— geometry

— Material.

$$P = f(F, G, M)$$

(functional, geometrical, material)

Performance metric $= f_1(F), f_2(G), f_3(M)$ (If all 3 aspects are independent)

for optimum design maximize P.

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$\sigma = \frac{My}{I}$

$\Rightarrow \sigma = \frac{M(32)}{\pi d^3}$

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Eg. Want to design a rod of circular cross section (solid), axially loaded.

Requirement : light, stiff.

$\Rightarrow$ Performance metric : Mass "m" to be minimized.

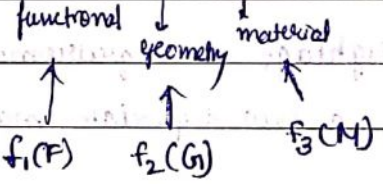
$$\text{stiffness} = \frac{F}{\delta} \qquad \delta = \frac{FL}{AE}$$

$$\Rightarrow \frac{F}{\delta} = \frac{AE}{L} = k$$

$$\Rightarrow \boxed{A = \frac{kL}{E}}$$

$$m = \rho AL = \frac{\rho k L^2}{E}$$

$$\Rightarrow m = (k)(l^2)\left(\frac{\rho}{E}\right)$$



Functional requirement = k

Geometrical aspect = l^2

Material aspect = $\frac{\rho}{E}$

∴ to minimize m

when material is only concern $\rightarrow$ $f_3(M)$ is called Material efficiency coefficient. In our case

In our case, $f_3(M)$ has to be ~~maximized~~ minimized

That is minimize $\frac{\rho}{E}$ or maximize $\frac{E}{\rho}$.

If k is performance metric,

$$k = \underset{F}{(m)}\left(\frac{1}{l^2}\right)\underset{G}{}\left(\frac{E}{\rho}\right)\underset{M}{}$$

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Sapphire — α Al_2O_3

18th Jan

Material selection: Choose material for crankshaft for air compressor

⇒ Application Requirements — Compact (strength/volume)

Probable failure modes — fatigue, wear, yielding

Cost is also a consideration

	Application requirement	Material property
(1) (key aspect)	Strength to volume (compactness)	yield / ultimate strength
(2)	Stiffness	Young's Modulus
(3)	Wear Resistance	Hardness
(4)	Cost constraint	Cost / Machinability

Select Material :

High strength to volume

- ✓ Ultra high strength steel
- High carbon steel
- Ceramic (strength in compression)
- Medium carbon steel
- stainless steel
- ✓ low carbon steel (Case Hardened)
- ✓ gray CI

Stiffness

- Tungsten carbide (W_2C)
- steel
- stainless steel
- Cast Iron

Hardness

- Diamond
- Tungsten Carbide
- ✓ Low Carbon steel (Case hardened)
- ✓ Ultra high strength steel
- ✓ gray cast iron

Material cost

- ✓ gray CI
- ✓ low carbon steel (Case Hard)
- ✓ Ultra high strength steel
- Acrylic
- stainless steel

	High strength steel	Low C steel	Gray cast Iron
σ_{ut} / S_{ut}	1977 MPa	420 MPa	345 MPa
σ_{yield}	1850 MPa	351 MPa	—

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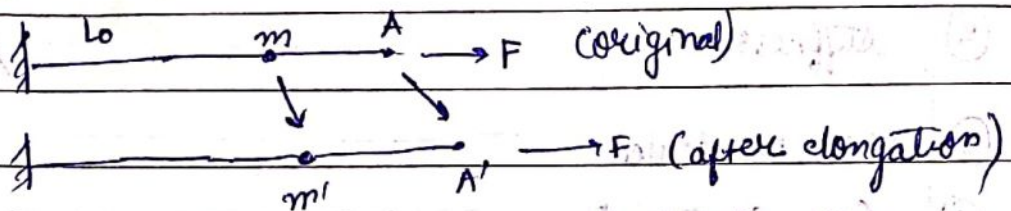
E	210 GPa	210 GPa	210 GPa
cost ₹/kg	100	75	45
Machinability Index	50	65	40
Hardness (Brinell)	560	650	262

23rd Jan. • Strain & Deformation



Initial length (original) = Lo final length = Lf

Point A goes to A'

—— m —— m'

Define strain :

(1) $e = \frac{L_f - L_o}{L_o} = \frac{\Delta u}{L_o}$: Engineering strain

(2) $\varepsilon = \frac{L_f - L_o}{L_f} = \frac{\Delta u}{L_f}$: True strain

(3) $\varepsilon = \ln\left(\frac{L_f}{L_o}\right) = \ln\lambda$: Logarithmic strain.

$\lambda = \frac{L_f}{L_o}$ (Stretch)

$= \ln\left(\frac{1 + \Delta u}{L_o}\right)$ → Taylor series

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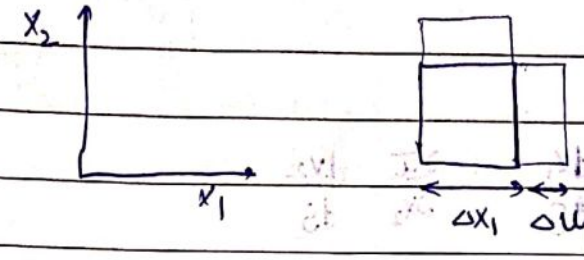
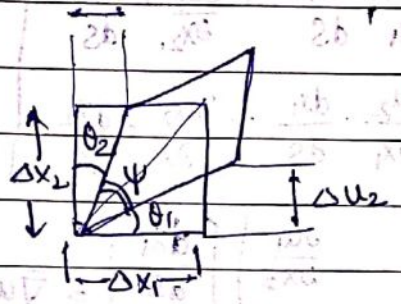
In 2D



Elongation along x_1	Elongation along x_2
$\varepsilon_{11} = \varepsilon_x \cong \frac{\Delta u_1}{\Delta x_1}$	$\varepsilon_y = \varepsilon_{22} \cong \frac{\Delta u_2}{\Delta x_2}$

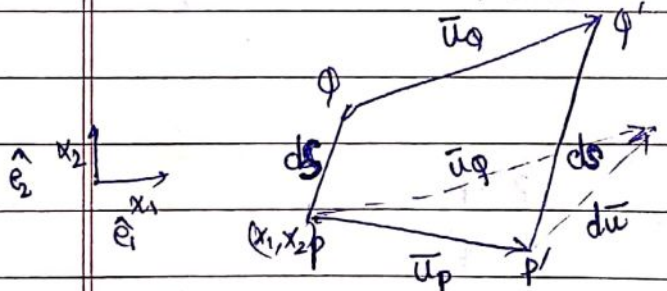


$$\gamma_{12} = \frac{\pi}{2} - \psi = \theta_1 + \theta_2$$

$$\cong \frac{\Delta u_2}{\Delta x_1} + \frac{\Delta u_1}{\Delta x_2} = 2\varepsilon_{12}$$

$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}$	$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2}$	$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$

- Material resists relative motion / displacement between points
- Any definition of strain must give a quantitative measurement of such relative displacement.



PQ → stretched & rotated

u_P = displacement of point P

u_Q = displacement of point Q.

$d\bar{u}$ = displacement of Q relative to P

$= \bar{u}_Q - \bar{u}_P$

$$\overrightarrow{PQ} = dx_1\hat{e}_1 + dx_2\hat{e}_2$$

$\bar{n}$ = unit vector along PQ $= n_1\hat{e}_1 + n_2\hat{e}_2$

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$$= \frac{dx_1}{ds}\hat{e}_1 + \frac{dx_2}{ds}\hat{e}_2$$

Look at:

$$\frac{d\bar{u}}{ds} = \frac{\partial \bar{u}}{\partial x_1}\cdot\frac{dx_1}{ds} + \frac{\partial \bar{u}}{\partial x_2}\cdot\frac{dx_2}{ds}$$

$$\frac{d\bar{u}}{ds} = \left(\frac{du_1}{ds}\right)\hat{e}_1 + \left(\frac{du_2}{ds}\right)\hat{e}_2 = \begin{bmatrix} du_1/ds \\ du_2/ds \end{bmatrix}$$

$$\Rightarrow \frac{d\bar{u}}{ds} = \begin{bmatrix} \frac{du_1}{ds} \\ \frac{du_2}{ds} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1}\cdot\frac{dx_1}{ds} + \frac{\partial u_1}{\partial x_2}\cdot\frac{dx_2}{ds} \\ \frac{\partial u_2}{\partial x_1}\cdot\frac{dx_1}{ds} + \frac{\partial u_2}{\partial x_2}\cdot\frac{dx_2}{ds} \end{bmatrix}$$

rotation + stretching

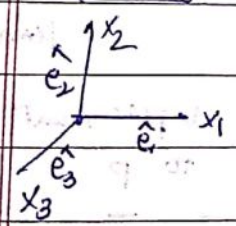
$$= \underbrace{\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{bmatrix}}_{\bar{\nabla}\bar{u}} \underbrace{\begin{bmatrix} \frac{dx_1}{ds} \\ \frac{dx_2}{ds} \end{bmatrix}}_{\bar{n}} = \bar{\nabla}\bar{u}\cdot\bar{n}$$

(Gradient of $\bar{u}$)

Strain Matrix: Now, let $\bar{\bar{E}} = \frac{1}{2}\left[\bar{\nabla}\bar{u} + (\bar{\nabla}\bar{u})^T\right]$ ← symmetric part of $\bar{\nabla}\bar{u}$

Rotation Matrix: $\bar{\bar{\Omega}} = \frac{1}{2}\left[\bar{\nabla}\bar{u} - (\bar{\nabla}\bar{u})^T\right]$ ← skew symmetric

In 3D



$P \to P'$

$Q \to Q'$

$d\bar{u} = \bar{u}_Q - \bar{u}_P = \begin{bmatrix} du_1 \\ du_2 \\ du_3 \end{bmatrix}$

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$$\frac{du_i}{ds} = \frac{\partial u_i}{\partial x_1}\frac{dx_1}{ds} + \frac{\partial u_i}{\partial x_2}\frac{dx_2}{ds} + \frac{\partial u_i}{\partial x_3}\frac{dx_3}{ds} \qquad i = 1,2,3$$

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{ds} \\ \frac{dx_2}{ds} \\ \frac{dx_3}{ds} \end{bmatrix}$$

Finally get, $\frac{d\bar{u}}{ds} = \bar{\nabla}\bar{u}\cdot\bar{n} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$

Stretching $\bar{\bar{E}}$ = symmetric part of $\bar{\nabla}\bar{u} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \\ & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) \\ \text{Symmetric} & & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$

Rotation $\bar{\bar{\Omega}}$ = skew $\bar{\nabla}\bar{u} = \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ \text{Skew Symm.} & & 0 \end{bmatrix}$

25th Jan. **Stresses**

Forces

① Body force – long range (gravity)

② ~~Contact force~~ Surface force – contact forces on the boundary of the body

("TRACTION FORCES") (pressure, friction)

– Force per unit area

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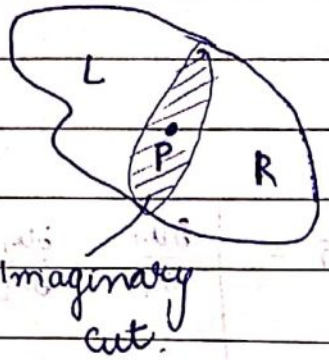
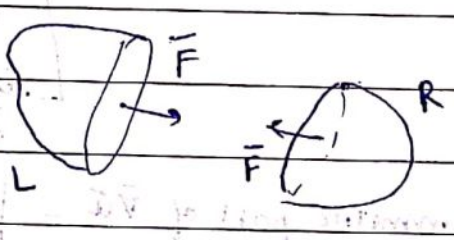
Cauchy Hypothesis : At any point inside a body - can define a surface traction field.

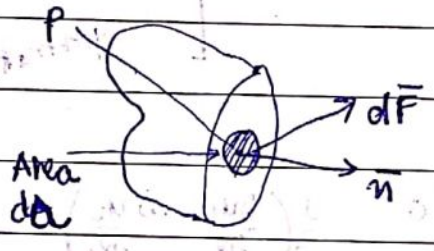


- Consider an imaginary cut through the body
- It creates additional surfaces.

Draw FBD of left and right.



Detailed diagram of left part.



Effect of the right part on the area da around P is represented by the force $d\bar{F}$ (resultant of the distributed force acting over da)

- $d\bar{F}$ can be thought of as a contact force
- define traction / stress vector

$$\bar{t} = \lim_{da \to 0} \frac{d\bar{F}}{da}$$

(traction vector at P)
Note : limit exists & P is inside da

For a point P for specified $\bar{n}$ $\therefore (\bar{x}, \bar{n})$ are fixed

- traction vector $\bar{t}$ (at P) depends on the orientation of the imaginary cut plane

$$\bar{t} = \bar{t}(\bar{x}, \bar{n}) \leftarrow \text{surface traction field (force per unit area)}$$

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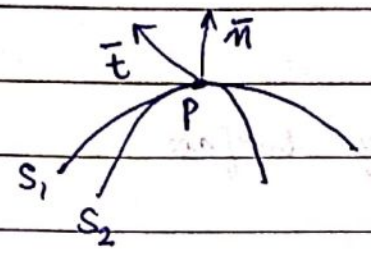
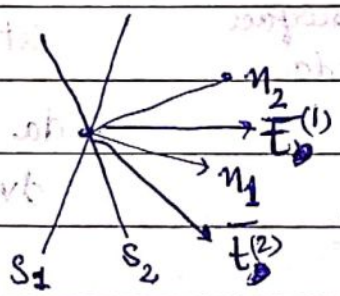
Scenario 1: $\bar{P}$ is fixed. Take different imaginary surfaces with same normal at $\bar{P}$

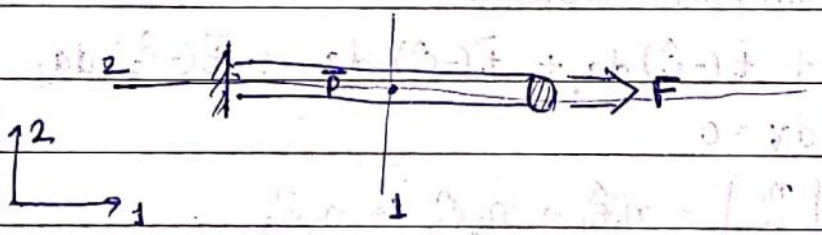


$\Rightarrow$ The traction vector will be same

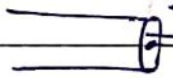
Scenario 2: Two surfaces pass through $\bar{P}$ with different normals



Example



1:

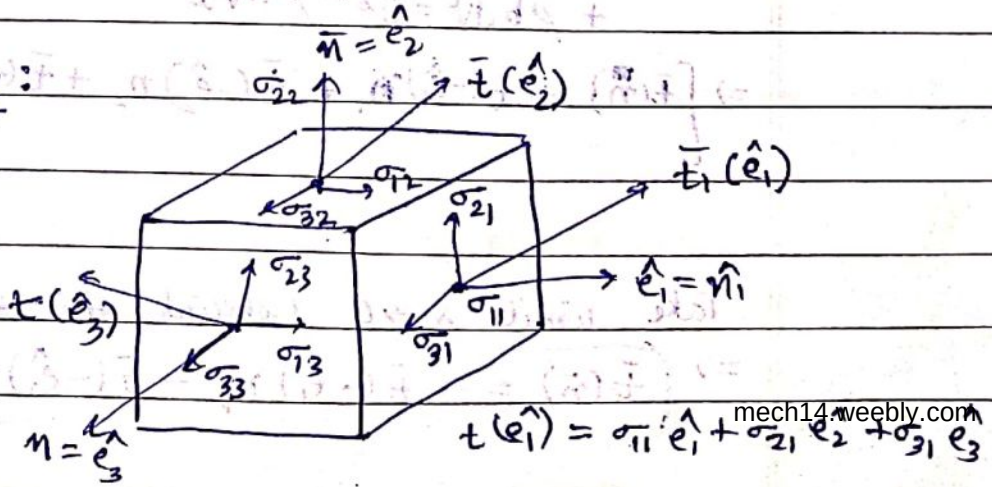


$$\bar{t}_1 = \frac{F}{A}\hat{e}_1$$

2:

$$\bar{t}_2 = \bar{0}$$

For 3D body:



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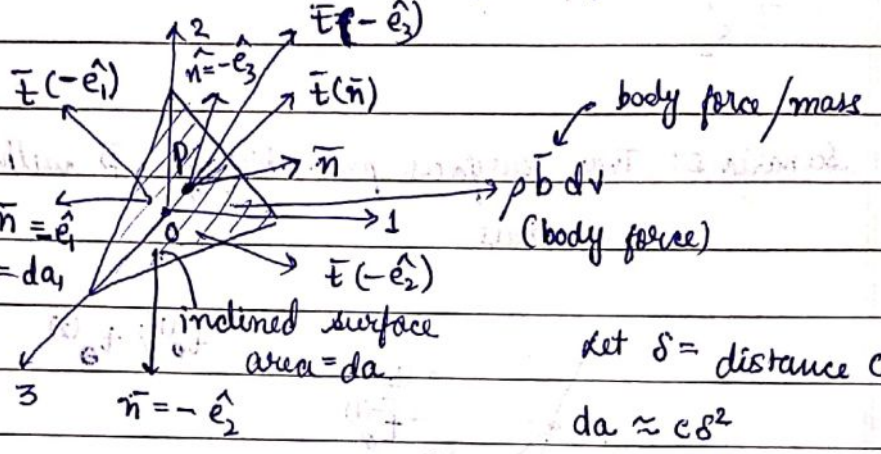
- Direction of traction vector is same as direction of force vector
- Only the magnitude of area changes $\left[\left(\frac{A}{\cos\theta}\right)\right]$

$$\bar{t} = \bar{t}(\hat{e}_1) = \begin{bmatrix} \sigma_{11} \\ \sigma_{21} \\ \sigma_{31} \end{bmatrix} \quad \text{or} \quad \bar{t}(\hat{e}_i) = \begin{bmatrix} \sigma_{1i} \\ \sigma_{2i} \\ \sigma_{3i} \end{bmatrix} \quad i = 1,2,3.$$

- Traction vector on arbitrary surface



Let δ = distance O to P

$da \approx c\delta^2$

$dv = k\delta^3$

Cauchy Tetrahedron.

Linear Momentum Balance:

$$\bar{t}(\bar{n})\, da + \bar{t}(-\hat{e}_1)\, da_1 + \bar{t}(-\hat{e}_2)\, da_2 + \bar{t}(-\hat{e}_3)\, da_3 + \rho \bar{b}\, dv = 0$$

$$\Rightarrow \bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = n_1\hat{e}_1 + n_2\hat{e}_2 + n_3\hat{e}_3$$

$$d\bar{a} = da\,\bar{n} = \begin{bmatrix} da\,n_1 \\ da\,n_2 \\ da\,n_3 \end{bmatrix} = \begin{bmatrix} da_1 \\ da_2 \\ da_3 \end{bmatrix}$$

$$\therefore t(\bar{n})\, da + \bar{t}(-\hat{e}_1)\, da\,n_1 + \bar{t}(-\hat{e}_2)\, da\,n_2 + \bar{t}(-\hat{e}_3)\, da\,n_3 + \rho \bar{b}\, dv = 0 \leftarrow \rho\, dv\, \bar{a}$$

$$\Rightarrow \left[\bar{t}(\bar{n}) + \bar{t}(-\hat{e}_1)\, n_1 + \bar{t}(-\hat{e}_2)\, n_2 + \bar{t}(-\hat{e}_3)\, n_3\right] + \rho \bar{b}\left(\frac{k}{c}\delta\right) = \rho \bar{a}\, \frac{k}{c}(\delta)$$

Take limit $\delta \to 0$ (shrink the tetrahedron)

$$\Rightarrow \boxed{\bar{t}(\bar{n}) = -\bar{t}(-\hat{e}_1)\, n_1 - \bar{t}(-\hat{e}_2)\, n_2 - \bar{t}(-\hat{e}_3)\, n_3}$$

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$\bar{t}(\bar{n})$ is a linear combination of traction vector of 3 $\perp^r$ planes.

Now let: $\bar{n} = \hat{e}_1 \Rightarrow n_1 = 1$

$n_2 = n_3 = 0$

$\Rightarrow t(\hat{e}_1) = -\bar{t}(-\hat{e}_1)$

Similarly $t(\hat{e}_2) = -t(-\hat{e}_2)$

$t(\hat{e}_3) = -t(-\hat{e}_3)$

$$\bar{t}(\bar{n}) = \bar{t}(\hat{e}_1)n_1 + \bar{t}(\hat{e}_2)n_2 + \bar{t}(\hat{e}_3)n_3.$$

Let $\bar{t}(\bar{n}) = t_1(\bar{n})\hat{e}_1 + t_2(\bar{n})\hat{e}_2 + t_3(\bar{n})\hat{e}_3$

$$\Rightarrow \begin{bmatrix} t_1(\bar{n}) \\ t_2(\bar{n}) \\ t_3(\bar{n}) \end{bmatrix} = \begin{bmatrix} \sigma_{11} \\ \sigma_{21} \\ \sigma_{31} \end{bmatrix} n_1 + \begin{bmatrix} \sigma_{12} \\ \sigma_{22} \\ \sigma_{32} \end{bmatrix} n_2 + \begin{bmatrix} \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \end{bmatrix} n_3$$

$$= \underbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}}_{\text{Matrix of stress components}} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

~~Then~~

then $\bar{t}(\bar{n}) = \bar{\bar{\sigma}} \cdot \bar{n}$

↑

stress matrix or stress tensor

Can show using angular momentum balance that,

$$\bar{\bar{\sigma}} = \bar{\bar{\sigma}}^T$$

i.e. $\bar{\bar{\sigma}}$ is symmetric $\Rightarrow$ six Unknown

$\sigma_{11}, \sigma_{22}, \sigma_{33}$ ← Normal stress components

$\sigma_{12}, \sigma_{13}, \sigma_{23}$ ← Shear stress components

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30th Jan

Failure theory:

Failure – A part gets broken / damaged / distorted → function compromised / not reliable; will discuss – strength / stress based failure.

Under static loading

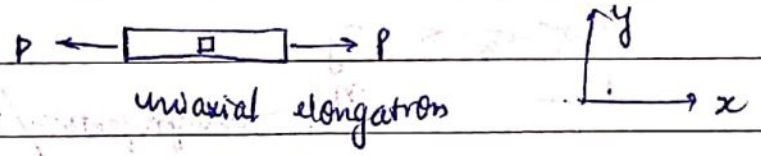
(1) Tresca failure criterion (Max. shear stress criterion)

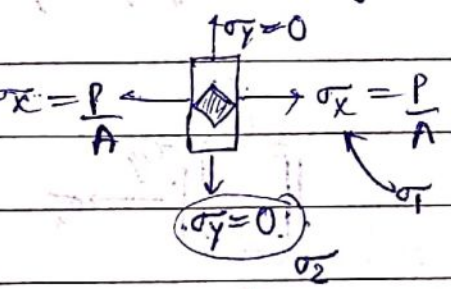
only for ductile material

slip along maxm. density planes

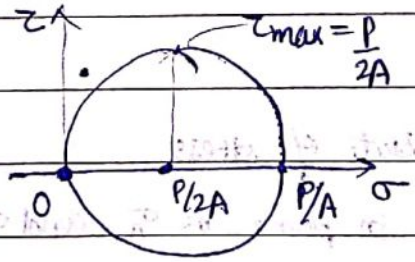
- Ductile materials undergo yielding before breaking

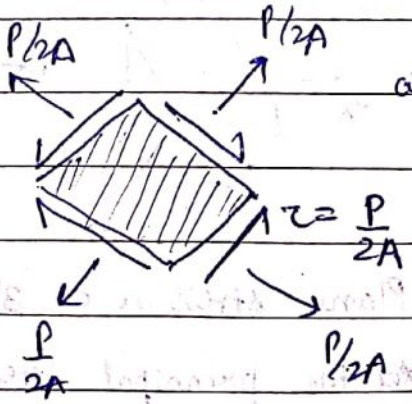
MSS Theory – failure / yielding happens when maximum shear stress reaches a critical value determined by the experiment.





Maxm shear stress $= \frac{\sigma_1 - \sigma_2}{2} = \frac{P}{2A}$





To avoid failure, $\tau_{max} = \frac{P}{2A} \leq$ shear strength

For 3D state of stress

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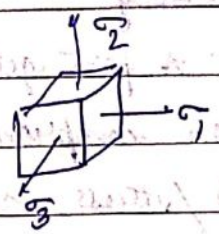
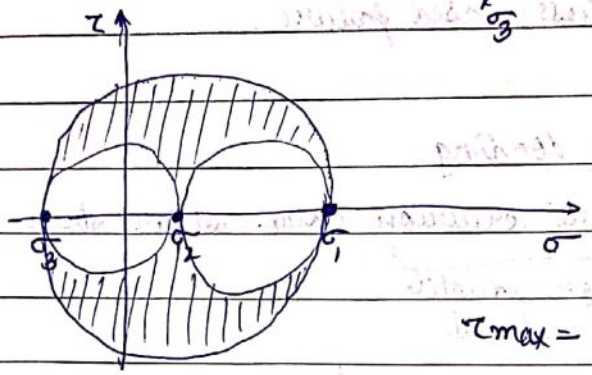
Symmetric matrix ⇒ 3 Real Eigen values

classmate
Date
Page

Principal stresses

$\sigma_1 \geq \sigma_2 \geq \sigma_3$





$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

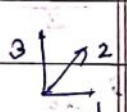
Yield strength in shear. $S_{sy} \geq \tau_{max}$ to avoid failu

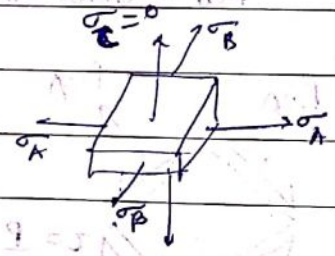
$$S_{sy} = \frac{S_y}{2_1}$$ ← Yield strength in tension

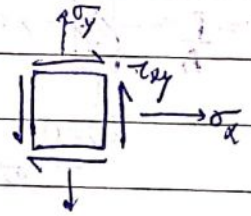
∴ MSS criterion. $(\sigma_1 - \sigma_3) \leq S_y$ to avoid failure

$$\sigma_1 - \sigma_3 \leq \frac{S_y}{n}$$ ← design factor / FOS

Plane stress



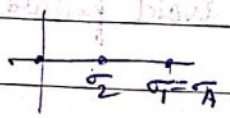




- Plane stress is a 3D state of stress
- Let the principal stresses in plane be σ_A and σ_B

  Principal stress out of plane $\sigma_3 = 0$

Scenario 1 $\sigma_A \geq \sigma_B > 0$



$$\tau_{max} = \frac{\sigma_A}{2}$$

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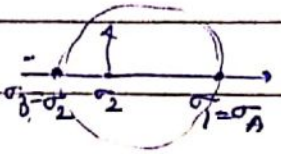
MSS : $\tau_{max} = \frac{\sigma_A}{2} \leq S_{sy} = \frac{S_y}{2}$

$\Rightarrow$ $\boxed{\sigma_A \leq S_y}$ to avoid failure.

Scenario 2 $\sigma_A \geq 0 \geq \sigma_B$



$\tau_{max} = \frac{\sigma_A - \sigma_B}{2} \leq S_{sy} = \frac{S_y}{2}$

$\Rightarrow$ $\boxed{(\sigma_A - \sigma_B) \leq S_y}$

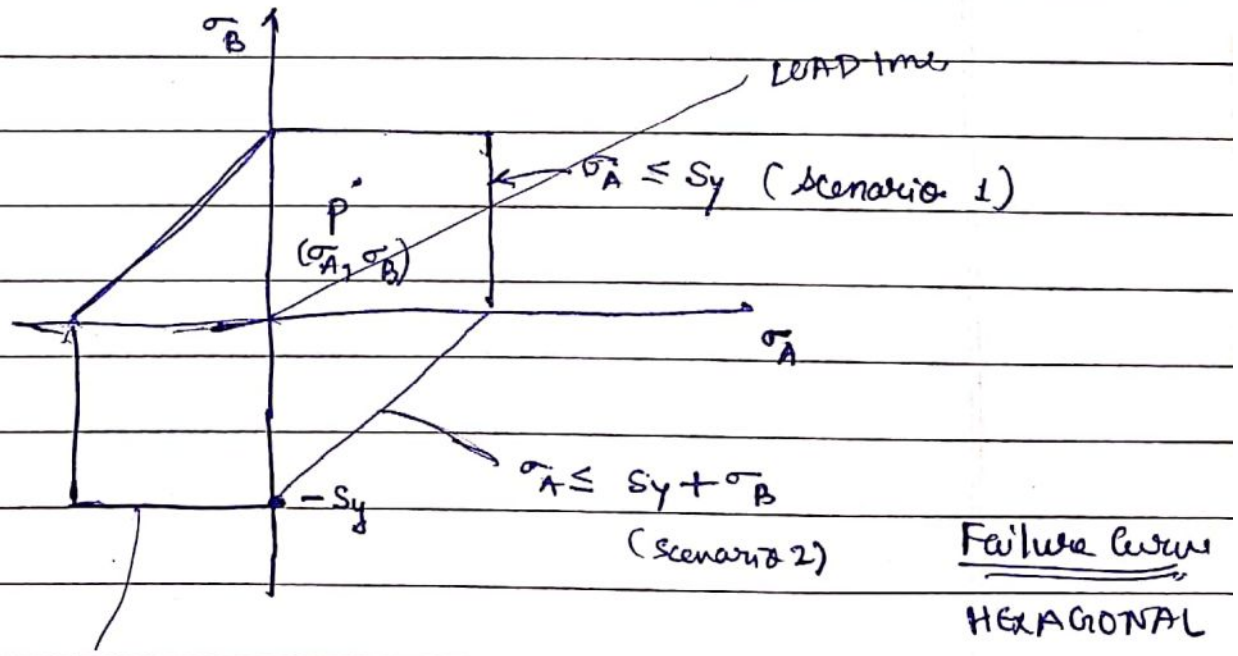
Scenario 3 $0 \geq \sigma_A \geq \sigma_B$

$\sigma_1 = 0$; $\sigma_2 = \sigma_A$; $\sigma_3 = \sigma_B$.

$\tau_{max} = \frac{-\sigma_B}{2}$

MSS. $-\sigma_B \leq S_y$

$\Rightarrow$ $\boxed{\sigma_B \geq -S_y}$



Failure curve

HEXAGONAL

TRESCA FAILURE ENVELOPE

To calculate FOS : LOAD line must not cross

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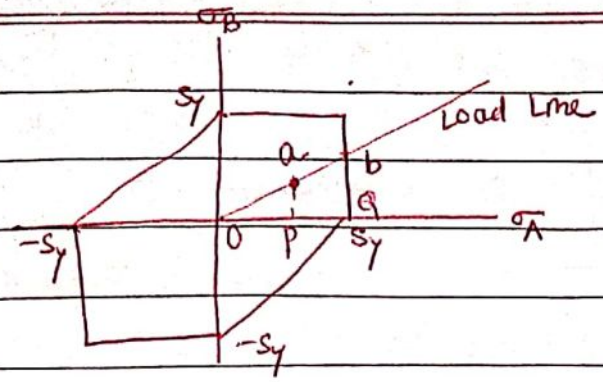


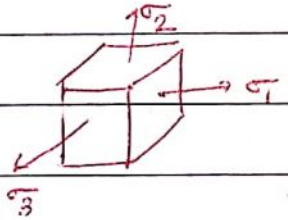
$$FOS = \frac{Ob}{Oa}$$

- For a given load scenario principal stresses are given by part a.
- With increasing load, stresses increase proportionately ⇒ gives a load line.
- Failure occurs when loadline intersects the failure envelope.

$$\boxed{FOS = n = \frac{Ob}{Oa} = \frac{OQ}{OP}} \quad (\triangle \text{ rule})$$

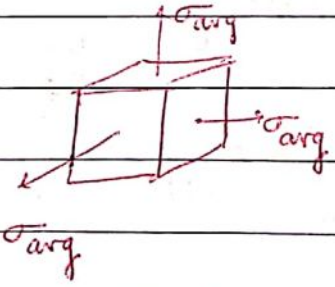
VON MISES YIELD CRITERIA

Yielding occurs when distortional strain energy reaches a critical value.



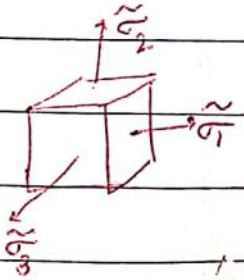
Principal coordinate system

$=$



Hydrostatic

$+$



Distort

$$\sigma_{avg} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Causes volume change only

$$\tilde{\sigma}_1 = \sigma_1 - \sigma_{avg}$$

$$\tilde{\sigma}_2 = \sigma_2 - \sigma_{avg}$$

$$\tilde{\sigma}_3 = \sigma_3 - \sigma_{avg}$$

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$$\underline{\tilde{\sigma}} = \begin{bmatrix} \sigma_1 - \sigma_{avg} & 0 & 0 \\ 0 & \sigma_2 - \sigma_{avg} & 0 \\ 0 & 0 & \sigma_3 - \sigma_{avg} \end{bmatrix}$$

$$I_1(\underline{\tilde{\sigma}}) = \tilde{\sigma}_1 + \tilde{\sigma}_2 + \tilde{\sigma}_3 = 0$$

causes shape distortion only.

No change in volume.

Total strain Energy :

$$u = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3)$$

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu(\sigma_2+\sigma_3)) \quad \Big| \quad \varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu(\sigma_1+\sigma_3))$$

$$\varepsilon_3 = \frac{1}{E}(\sigma_3 - \nu(\sigma_1+\sigma_2))$$

$$\therefore u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Strain energy due to hydrostatic part, let $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_{avg}$ in eq (1).

$$\Rightarrow u_v = \frac{3}{2E}(1-2\nu)\sigma_{avg}^2 = \frac{(1-2\nu)}{6E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)^2$$

$$u = u_v + u_d$$

u_v ↑ hydrostatic, u_d ↖ distortional

$$u_d = u - u_v = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right)$$

$$- \frac{(1-2\nu)}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$$

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$$\Rightarrow u_d = \frac{(1+\nu)}{6E}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

Von-Mises Criteria

$$u_d = \frac{(1+\nu)}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \leq \text{critical value}$$

(critical value determined from experiment to avoid failure)

Case 1: simple Elongation. $\sigma_2=\sigma_3=0$ (failed when $\sigma_1 = S_y$)

$$u_d = \frac{1+\nu}{3E}\sigma_1^2 \leq \frac{1+\nu}{3E}S_y^2$$

$\Rightarrow \sigma_1 \leq S_y$ to avoid failure

In the general scenario, critical value of distortional energy $\frac{1+\nu}{3E}S_y^2$

Failure Criteria $\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \leq S_y$

σ' or Von Mises stress

Plane stress

$\sigma_1 = \sigma_A$; $\sigma_2 = \sigma_B$; $\sigma_3 = 0$

Von Mises stress

$$\sigma' = \left(\sigma_A^2+\sigma_B^2-\sigma_A\sigma_B\right)^{1/2} \leq S_y$$ to avoid failure.

Failure curve is an ellipse in $\sigma_A - \sigma_B$ plane.

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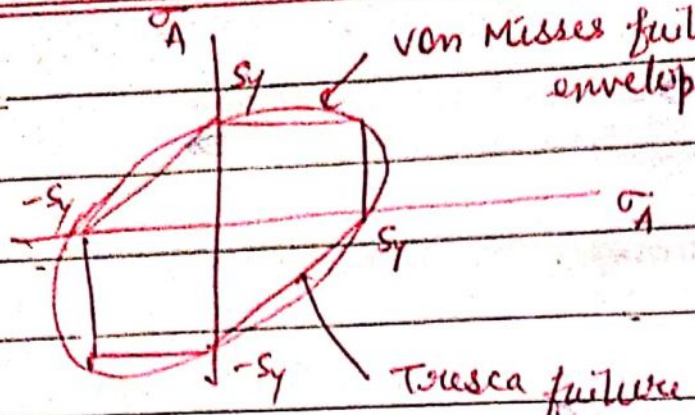

∴ Tresca is more conservative.

Factor of safety

Tresca ; $n = \frac{Ob_1}{Oa}$, von Misses $= \frac{Ob_2}{Oa} > \frac{Ob_1}{Oa}$

For general state of stress,

$$\underline{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

~~$\sigma' = \frac{1}{\sqrt{2}} \left[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right]^{1/2}$~~

$$\sigma' = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right]^{1/2}$$

For plane stress $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

$$\sigma' = \left[\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 \right]^{1/2}$$

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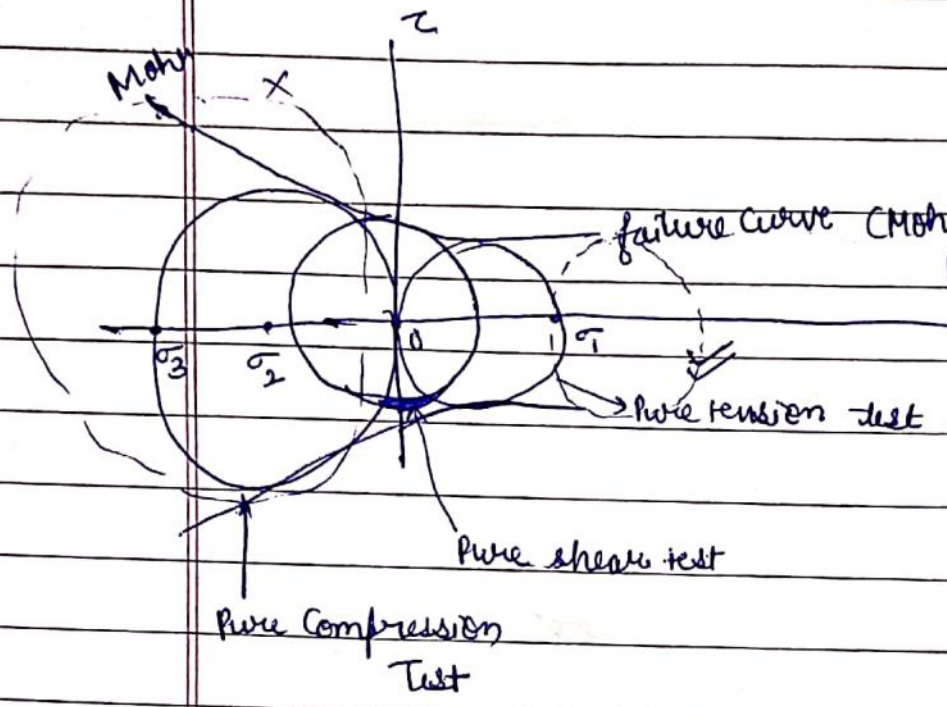
8th Feb

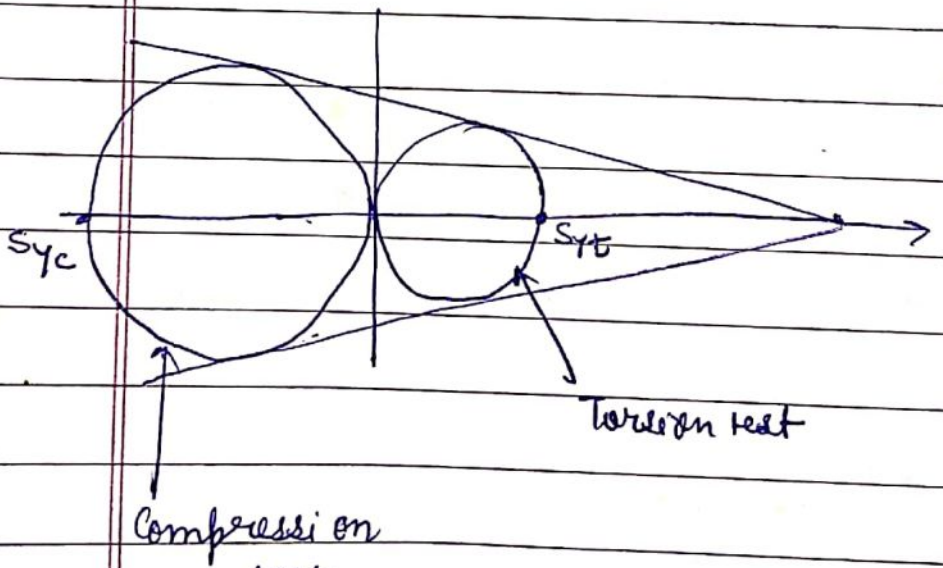
Von Mises & Tresca – Yield strength in tension and compression were taken to be same.

When they are different (eg Mg. alloy) Mohr or Mohr-Coulomb criteria are used (for ductile material)



Mohr-Coulomb: Assumes failure curve is straight

Two tests are needed — tension & compression



☆ Particular 3D state of stress,

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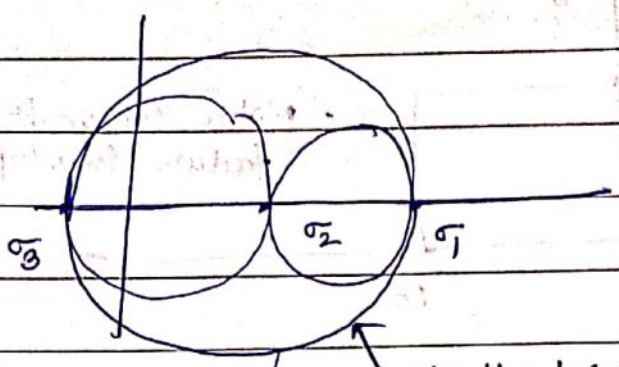
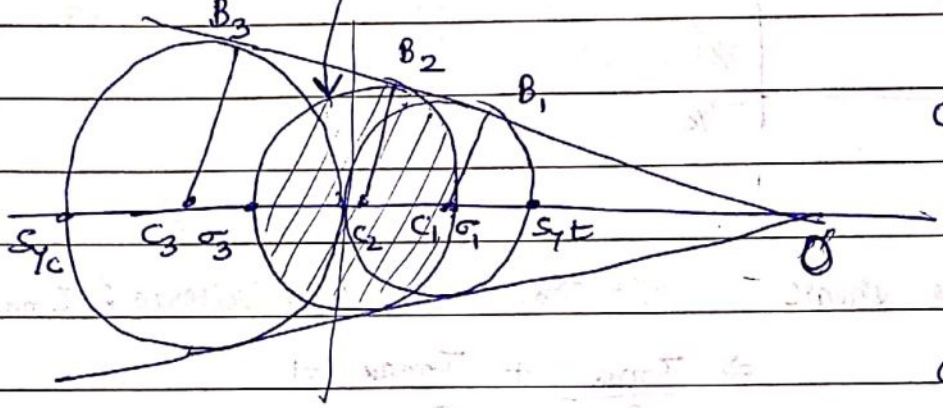
$\sigma_1 \geq \sigma_2 \geq \sigma_3$ principal.



use the largest circle for failure analysis



$C_1 = \frac{S_{yt}}{2}$

$C_2 = \frac{\sigma_1 + \sigma_3}{2}$

$C_3 = -\frac{S_{yc}}{2}$

Triangles ΔOB_1C_1, ΔB_2C_2, ΔOB_3C_3 are similar

$$\Rightarrow \frac{B_3C_3}{OC_3} = \frac{B_2C_2}{OC_2} = \frac{B_1C_1}{OC_1}$$

$$\Rightarrow \frac{B_3C_3 - B_1C_1}{OC_3 - OC_1} = \frac{B_2C_2 - B_1C_1}{OC_2 - OC_1}$$

($B_3C_3 = S_{yt}/2$, $B_1C_1 = S_{yt}/2$, $B_2C_2 = S_{yt}/2$; $OC_3 - OC_1 = C_1C_3$; $OC_2 - OC_1 = C_1C_2$)

$$\Rightarrow \frac{S_{yc} - S_{yt}}{S_{yc} + S_{yt}} = \frac{(\sigma_1 - \sigma_3) - S_{yt}}{S_{yt} - (\sigma_1 + \sigma_3)}$$

$$\Rightarrow \frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} = 1 \Rightarrow \boxed{\frac{\sigma_1}{S_{yt}} - \frac{\sigma_3}{S_{yc}} = \frac{1}{n} \leftarrow FOS}$$

$\boxed{S_{yt} = S_{yc} \equiv \text{TRESCA} = S_y}$

For Plane stress :

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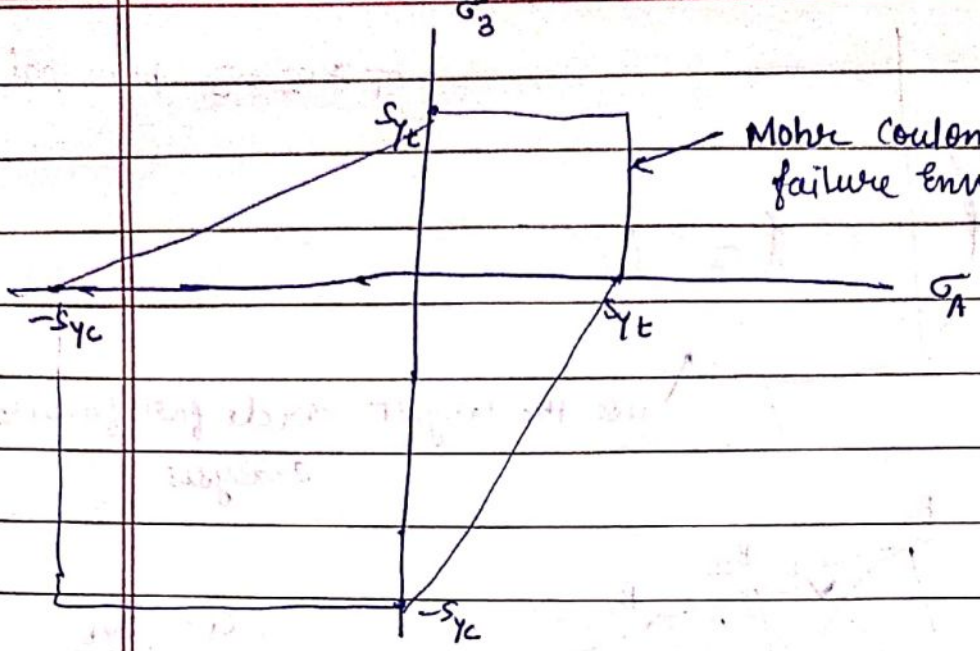

Pure shear $\sigma_1 = -\sigma_3 = \tau$, Yield Criteria : $\tau_{max} = S_{sy}$

$$\Rightarrow \frac{\tau_{max}}{S_{yt}} + \frac{\tau_{max}}{S_{yc}} = 1$$

$$\Rightarrow \frac{S_{sy}}{S_{yt}} + \frac{S_{sy}}{S_{yc}} = 1$$

$$\Rightarrow \boxed{S_{sy} = \frac{S_{yt} \cdot S_{yc}}{S_{yt} + S_{yc}}}$$

Failure Criteria – Ductile Mats.

(1) If $S_{yc} = S_{yt}$, use von Mises or Tresca

(2) If $S_{yc} \neq S_{yt}$, use Mohr-Coulomb

Example A material has T.S $S_y = 650$ MPa. both in tension & compression.

Loading scenario, $\sigma_1 = 490$ MPa, $\sigma_2 = 210$ MPa, $\sigma_3 = 0$ MPa. (Plane stress)

Estimate FOS using Tresca & VON MISES

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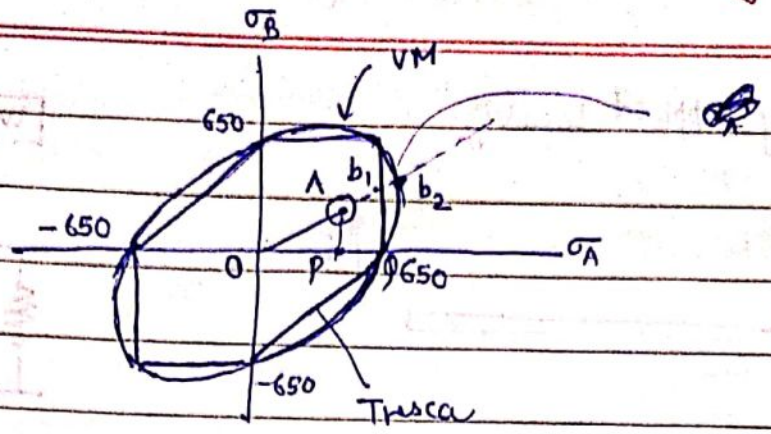
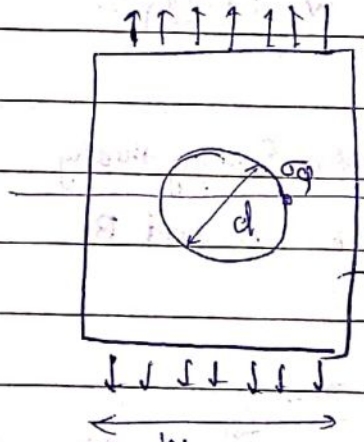


FOS Tresca $= \frac{Ob_1}{OA} = \frac{Oq}{OP} = \frac{650}{490} = 1.326$

Von-Mises $\sigma' = \sqrt{\sigma_A^2 + \sigma_B^2 - \sigma_A \sigma_B}$

$= 426$ MPa

- Stress Concentration.



$\sigma_A = \sigma_p (W-d) t$

$\Rightarrow \sigma_p = \frac{\sigma_A}{(W-d)t} = \frac{F}{(W-d)t}$

$\sigma_{max} = \sigma_q = \frac{3}{2} \frac{F}{(W-d)t}$

$= \left(\frac{3}{2}\right)(\sigma_p)$ → nominal stress.

K_t

↗ stress concentration factor in tension

K_t may depend on size of the hole, loading type

K_s = SCF in shear.

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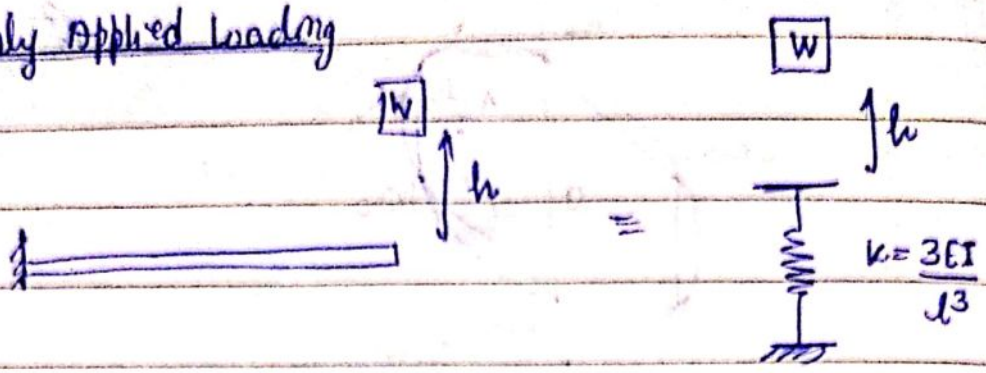
Suddenly Applied loading



Max^m deflection in beam = δ

∴ loss of PE @ Max defⁿ = (δ+h) W

Energy stored in spring @ Max^m defⁿ = $\frac{k\delta^2}{2}$

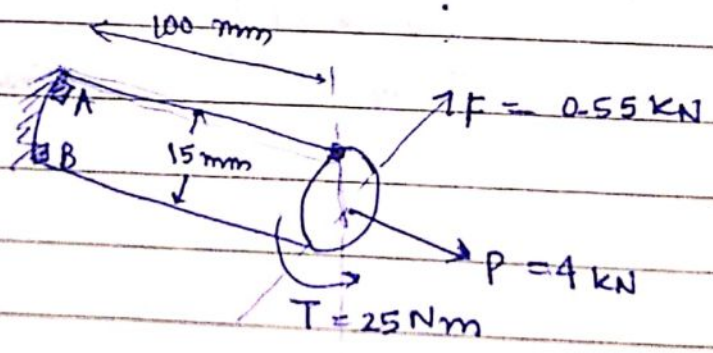
$\therefore \frac{k\delta^2}{2} = (\delta+h)W$

$\Rightarrow \delta = \frac{W}{k} \pm \frac{W}{k}\left(1+\frac{2hk}{W}\right)^{1/2}$

$\therefore \delta = \frac{W}{k} + \frac{W}{k}\left(1+\frac{2hk}{W}\right)^{1/2}$ (-ve not considered)

9th Feb FOS using Distortion Energy theory.

Two points given A and B



Material AISI 1006 CD

S_y = 280 MPa

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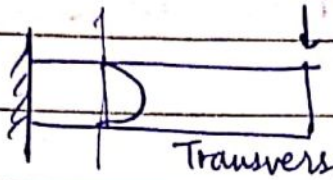
Point A Max Bending stress $\sigma_b = 0$

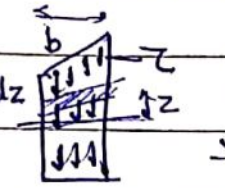
Max shear stress $\tau = \frac{16T}{\pi d^3}$ (for Torsion)

$37.72 + \frac{4F}{3A}$ (transverse load F)



Axial : $\frac{P}{A} = \frac{4P}{\pi d^2}$

$\tau_{Transverse} =$



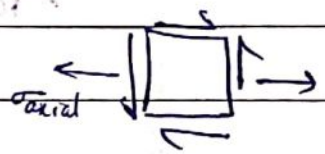
$\int_{-h}^{h} \tau b\,dz = V$

$\therefore \tau = 41.87$ MPa

$\sigma_{axial} = 22.6$ MPa

$\tau = \frac{VQ}{Ib}$

~~Point B~~ $\sigma' = (\sigma_x^2 + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}$



$\sigma' =$ ~~$(41.87^2 + 3 \times$~~ $(22.6^2 + 3 \times 41.87^2)^{1/2}$

$= 76$ MPa FOS $= 3.684$

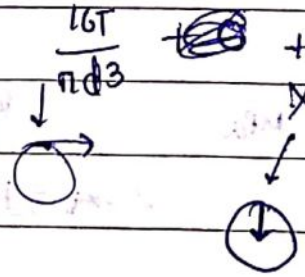
Point B Bending stress $\sigma_b = \frac{32M}{\pi d^3} = 165.9$ MPa

$\sigma_{axial} = 22.6$ MPa

} both added (tensile)

$\therefore \sigma_{Total} = \sigma_x = 188.6$ MPa

shear stress =



$= 37.8$ MPa

$\sigma' = (188.6^2 + 3 \times 37.8^2)^{1/2} = 200$ MPa

$\therefore$ FOS $= 1.4$ (Critical Point)

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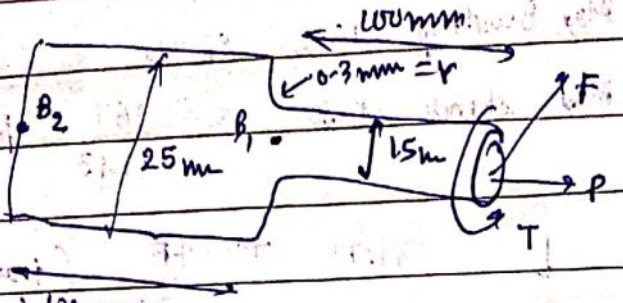

B_2

$$\sigma_B = \frac{32M}{\pi d^3} = \frac{32 \times (550 \times 0.15)}{\pi \times 25^3} \times 10^9 \text{ Pa} = 53.70 \text{ MPa}$$

~~For FOS,~~

B_1 is critical.

∴ For FOS, Account for stress con^n factor at B_1

$$\frac{D}{d} = \frac{25}{15} = \frac{5}{3} = 1.67$$

$$\frac{r}{d} = \frac{3}{15} = \frac{1}{5} = 0.2$$

Bending $\sigma_0 = 166$ MPa $\quad K_t = 1.41$

M ∴ 234 MPa

Axial $\sigma_0 = 22.6$ MPa $\quad K_t = 1.6$. (Shigley)

$\sigma_{axial} = 1.6 \times 22.6$

$= 36.16$

Torsion $\tau_0 = 37.8$ MPa $\quad K_t = 1.22$ 46.116

~~∴ $K_t = 1.6$ (from Shigley)~~

∴ Von Mises

$$\sigma' = \sqrt{(36.1 + 234)^2 + 3 \times 46.1^2} = 281.65$$

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$\therefore$ POS = 0.99.

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