INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 17.02.2016(AN) Time: 2hrs
Mid Semester Examination (Spring) Full Marks:60
Subject No. ME41606 Students of BTech course
Subject Name: FINITE ELEMENT METHODS IN ENGINEERING
No. of students: 57
Instructions: ***Answer all Three questions, Assume suitable data, if required.***

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1. Finite Element formulation may be carried out using *weighted-residual methods* or by using *weak formulation*.
   - (a) Considering one dimension second order differential equation, explain the requirements that the interpolation functions should satisfy which are used for the above two methods respectively.
   - (b) Write the polynomials of least possible order in one-dimension (function of x) for both the methods, if u is the field variable and also mention the names of the interpolation functions to be derived from the polynomials.
   - (c) What should be the nature of solution for u and its first derivative for the said interpolation functions at the nodal points? ***(15)***

2. The governing equation for the stepped shaft problem (Fig.1) with usual notations is given below:

$$-\frac{d}{dx}\left(EA\frac{du}{dx}\right)=0 \quad \text{for } 0<x<L \qquad \text{with boundary conditions, } u(0)=u(L)=0$$

   - (a) Derive the Rayleigh- Ritz finite element formulation.
   - (b) Compute the nodal displacements using three linear elements. You may assume the K-matrix.
   - (c) Compute the end reactions using (i) original matrix and (ii) post processing definition. Give your comments on the results obtained. ***(25)***

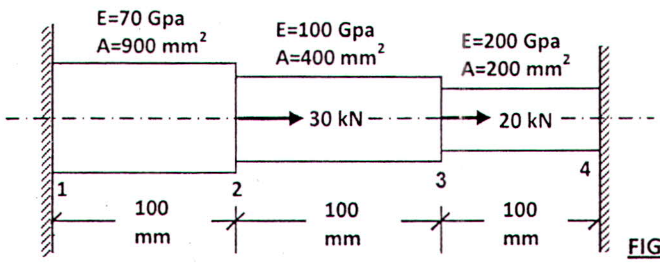

FIG.1

3. The velocity distribution u(y) for parallel flow between two long flat plates, separated by a distance of 2L and for a constant pressure gradient of dP/dx= q_0 may be obtained by solving the following differential equation:

$$-\frac{d}{dy}\left(\mu\frac{du}{dy}\right)+q_0=0 \quad \text{for } -L<y<L \qquad \text{with boundary conditions, } u(-L)=u(L)=0$$

   - (a) Compute u at the nodes using one quadratic element. You need not derive the weak form and interpolation functions; but you need to derive the [K] and {q} matrices.
   - (b) Derive the expression for exact solution of u (y).
   - (c) Compare the finite element solution with exact solution at y= - L, - L/2, 0, L/2 and L. ***(20)***

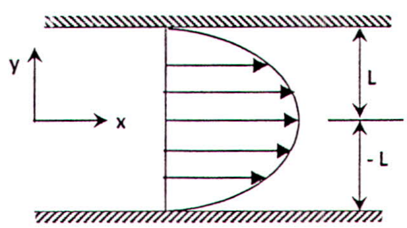

FIG.2