INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date of Examination: 19.04.2018(AN)
End Semester Examination (spring)
Subject No: ME41606
No. of students: 58

Time: 3hrs
Maximum Marks: 100
Subject Name: FINITE ELEMENT METHODS IN ENGINEERING

Instructions: *Answer all FIVE questions. Assume suitable data, if required.*

---

1. What are the conditions that an interpolation function should satisfy and also state its properties? Explain with examples, what are C^0 and C^1 elements? Derive interpolation functions for eight node Serendipity element.
(6+4+5)

2. The members of the truss, shown in Fig.1, are joined with frictionless pins. The area and the length of each member is 500 mm^2 and 2000 mm respectively and E=200Gpa. The truss is supported by a hinge joint at A and a roller joint at B.
   (a) Determine the nodal displacements in global coordinates(x and y) using FE analysis. Consider the connectivity of the elements as, **1** :(1-2)(1-2), **2** :(1-2)(1-3), **3**:(1-2)(2-3).
   (b) Find the normal stress in the member **3(2-3)** using the results of (a).
   (c) Find the normal stress in the member **3(2-3)** using the method of joints.
(12+8+5)

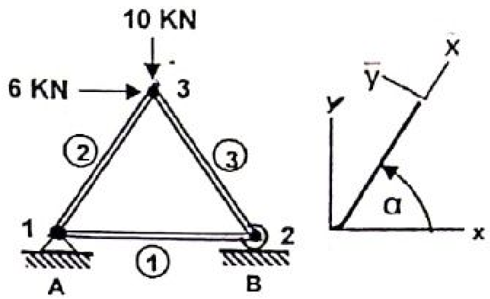

FIG. 1

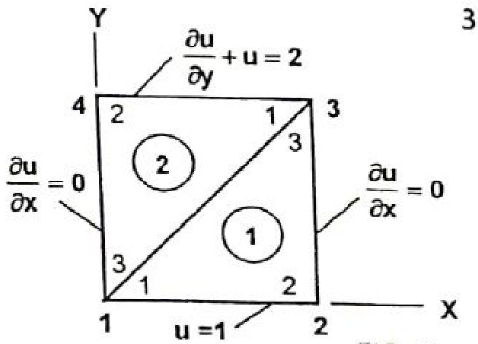

FIG. 2

3. Solve the Poisson's equation $-\nabla^2 u = 2$ for the square domain of sides 1 unit, as shown in Fig.2. The boundary conditions are, $u1 = u2 = 1$ on side 1-2, $\partial u/\partial n = 0$ on sides, 1-4, 2-3 respectively and $\partial u/\partial y + u = 2$ on side 3 - 4.
   (a) Derive the weak form of the differential equation and finite element formulation using linear triangular element.
   (b) Compute local and global [K] and {F} matrices.
   (c) Determine the nodal values of u.
(8+12+5)

4. A linear rectangular element in global coordinates (x,y) is shown in Fig.3. Derive expressions for $\frac{\partial \varphi_1}{\partial x}$, $\frac{\partial \varphi_1}{\partial y}$ and also compute the values at $\xi = \eta = 0$, using *isoparametric transformation*. φ_is (i=1- 4) are the interpolation functions in natural coordinates (ξ, η) of the linear rectangular element.
(15)

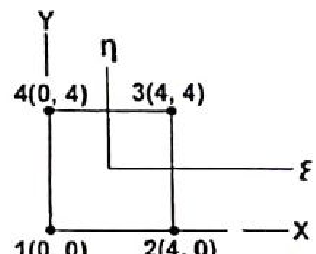

FIG.3

5. Solve the following differential equation,
$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 2 \quad \text{for } 0 < x < 1 \text{, with boundary conditions, } \frac{\partial u}{\partial x}(1,t) = 0 \quad \text{and} \quad u(0,t) = 0.0$$
and initial conditions, $u(x,0) = 0.0$.
   (a) Derive the semi-discrete finite element formulation.
   (b) Use α- family of time approximation (equation given) to compute the nodal values of {u} for two time steps only for $\alpha = 1/2$ and $\Delta t = 0.05$

   For the purpose of above computation use *two linear 1-D elements*. You may use standard forms of required matrices.
(10+ 15)

Useful Information

(a) For linear triangular element

$\varphi_i = 1/2A(\alpha_i + \beta_i x + \gamma_i y)$ $\quad 2A = \alpha_1 + \alpha_2 + \alpha_3$

where, $\alpha_i = x_j y_k - x_k y_j$

$\beta_i = y_j - y_k$

$\gamma_i = -(x_j - x_k)$

(b) Integration formula for triangle,

$$\int \varphi_1^m \varphi_2^n \varphi_3^p \, dxdy = \frac{m!n!p!}{(m+n+p+2)!} 2A$$

(c) Element stiffness matrix in global co-ordinates for truss

$$[K] = C \begin{bmatrix} C^2\alpha & & \text{symmetric} & \\ S\alpha C\alpha & S^2\alpha & & \\ -C^2\alpha & -S\alpha C\alpha & C^2\alpha & \\ -S\alpha C\alpha & -S^2\alpha & S\alpha C\alpha & S^2\alpha \end{bmatrix}$$

where, $C = EA/h$, $C\alpha = \text{Cos}\alpha$; $S\alpha = \text{Sin}\alpha$ and α is measured as shown in Fig.1

(d) 1-D Linear interpolation functions in natural co-ordinates

$\psi_1 = \frac{1}{2}(1-\xi)$, $\psi_2 = \frac{1}{2}(1+\xi)$

(e) α- family of time approximation (with usual notations),

$$[[M] + \alpha \Delta t [K]]_{S+1} \{u\}_{S+1} = [[M] - (1-\alpha)\Delta t [K]_S]\{u\}_S + \{\Delta t [\alpha \{F\}_{S+1} + (1-\alpha)\{F\}_S]\}.$$