Mid-Spring Semester Examination, 2017
Mechanical Engineering Department
Indian Institute of Technology, Kharagpur
Subject: Micromechanics and Nanomechanics (ME 60432)

Full Marks: 30 | Time : 2 Hrs

Answer all questions. Assume reasonably wherever it is necessary and all symbols are self-explanatory.

1. Show that the Green's function $G^{\infty}_{ij}(\hat{x}-\hat{y})$ of an unbounded elastic medium is given by

$$G^{\infty}_{ij}(\hat{x}-\hat{y}) = \int_{-\infty}^{\infty} \frac{1}{(2\pi)^3} \left(K_{ij}\right)^{-1} e^{i\hat{\varsigma}\cdot(\hat{x}-\hat{y})} d\hat{\varsigma}$$

where $\hat{\varsigma}$ is the position vector of a point in the medium in Fourier space and, $\hat{x}$ and $\hat{y}$ denote the same of the points in Euclidian space. (5)

2. (a) What is called eigenstrain? State the difference between an inclusion and an inhomogeneity. (2)

(b) Consider an unbounded elastic body in which an inclusion is present. The inclusion has got uniform eigenstrain ϵ^{*}_{mn}. Prove that the distribution of strain field (ϵ_{ij}) in the inclusion zone can be expressed as

$$\epsilon_{ij} = S_{ijmn}\ \epsilon^{*}_{mn}$$

where the Eshelby tensor S_{ijmn} is given by

$$S_{ijmn} = -\frac{1}{2}\int_{-\infty}^{\infty} C_{klmn}\{G^{\infty}_{ik,lj}(\hat{x}-\hat{y}) + G^{\infty}_{jk,li}(\hat{x}-\hat{y})\}d\hat{y}.$$ (5)

3. Consider a body with volume V and boundary S. Prove the Hill's Lemma as given below:

$$\overline{\sigma_{ij}\ \epsilon_{ij}} - \bar{\sigma}_{ij}\ \bar{\epsilon}_{ij} = \frac{1}{V}\int_S (u_i - x_j\ \bar{\epsilon}_{ij})(\sigma_{ik}n_k - \bar{\sigma}_{ik}n_k)dS$$

in which the symbols are self explanatory. (8)

4. (a) Consider a linear elastic composite body consisting of N different inhomogeneities and it is subjected to the homogeneous displacement boundary conditions on its surface. Show that the effective elastic coefficient matrix $[C]$ of the composite can be derived in terms of the concentration tensor $[A_r]$ of the r-th inhomogeneity as follows:

$$[C] = [C_0] + \sum_{r=1}^{N} v_r([C_r] - [C_0])[A_r]$$

where $[C_0]$ and $[C_r]$ are the elastic coefficient matrices of the matrix phase and the r-th inhomogeneity, respectively and v_r is the volume fraction of the r-th inhomogeneity. (5)

(b) In case of Mori-Tanaka micromechanics model, derive the expression for $[A_r]$. (5)