You may use the data:
1 barn = 10^{-24} cm^2, Avogadro number $N_0 = 6.023 \times 10^{23}$ mole^{-1},
1 eV = 1.602×10^{-19} J

1. A reactor core is in the form of an infinite slab, i.e., it has finite thickness in x-direction, while it is infinite in the y- and z-directions. The material buckling is B^2. What should be thickness of the slab for the reactor to be critical? If the reactor core has the form of a cylinder of infinite length what should be the radius for the reactor to be critical? Compare the dimension of the slab with that for a cube. Which is larger and why? Compare the radius of the infinite cylinder with that for a finite cylinder.

2. A nuclear reactor is to be designed to operate using uranium dioxide (containing natural uranium) as fuel and heavy water as moderator (with 325 molecules of D_2O for every atom of uranium). Assuming a homogeneous reactor model the infinite multiplication factor k_∞ is found to be 1.167. For pure heavy water we have $L^2 = 3 \times 10^4$ cm^2 and $L_s^2 = 131$ cm^2 where L and L_s are the diffusion and slowing down length respectively. For the given moderator fuel ratio we have $f = 0.9560$. Using modified one-group theory calculate the material buckling B_m^2. If the reactor core is to be designed in the shape of a cube solve the reactor equation and calculate what should be the dimension for the reactor to be critical? Assume that the extrapolation distance can be neglected compared to the dimensions of the reactor core. If a reactor core is designed in the shape of a rectangular parallelepiped of sides a, b and c prove that the smallest critical volume is obtained for a cube.

3. (a) A nuclear reactor uses natural uranium in the form of uranium dioxide UO_2 as fuel. The moderator used is heavy water D_2O while the coolant is light water H_2O. The reactor core contains 300 molecules of D_2O and 0.3 molecules of H_2O for every molecule of UO_2. Assuming a homogeneous reactor model calculate the value of the infinite multiplication factor k_∞. The resonance escape probability is given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N\left({}^{235}U\right)}\right\}^{-0.514}\right]$$

where $\Sigma_s/N({}^{238}U)$ is in barns. Assume that the fast fission factor $\epsilon \approx 1$. For fission of ^{235}U with thermal neutrons $\nu = 2.42$. For thermal neutrons:

^{238}U: $\sigma_c = 2.72$, $\sigma_f = 0$
^{235}U: $\sigma_c = 101$, $\sigma_f = 579$
O: $\sigma_a = 0$
D_2O: $\sigma_a = 0.001$
H_2O: $\sigma_a = 0.66$

The scattering cross-sections and values of ξ are:

U: $\sigma_s = 8.3$, $\xi = 0.0084$
O: $\sigma_s = 3.8$, $\xi = 0.1209$
D_2O: $\sigma_s = 10.6$, $\xi = 0.509$
H_2O: $\sigma_s = 50$, $\xi = 0.920$

All cross-sections are in barns. Natural uranium contains 0.715 percent ^{235}U (molal ratio).

(b) With the core composition given in part (a) a nuclear reactor is to be built with a spherical core. Solve the reactor equation in spherical coordinates

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) + B^2\phi = 0$$

and compute the radius of the core required for the reactor to be critical. Assume that $M^2 = 0.1985$ m^2.

4. (a) The core of a light water reactor is to be constructed in the shape of a rectangular parallelepiped of dimensions $H \times 1.2H \times 2H$. The composition of the core is uniform with $k_\infty = 1.370$ and $f = 0.897$. For light water $L^2 = 8.1$ cm^2 and $L_s^2 = 27$cm^2. What is the value of H required for the reactor to be critical? How much reduction in total volume of the core can be obtained by using a cubical shape?

   (b) The power generated per unit volume varies with position in the core. For a cubical core what is the maximum to average power generation per unit volume?

5. A reactor core is designed in the shape of a cube. The value of k_∞ is 1.28 and the square of migration length $M^2 = 35$ cm^2. What should be the length of each side of the cube for the reactor to be critical? If the macroscopic fission cross-section in the core is 0.1076 cm^{-1} and the neutron flux at the centre of the cube is 10^{14} neutrons/cm^2 sec what is the total rate of thermal energy generated by fission in the reactor in MW? The energy from fission of one nucleus is 200 MeV. Assume uniform composition in the entire core and neglect the extrapolation distance.

6. A nuclear reactor core contains uranium dioxide, UO_2, and light water, H_2O, in the ratio 1:4 (1 molecule of fuel to 4 molecules of moderator). The uranium in UO_2 contains 3 percent ^{235}U. It can be shown that with this composition of the core $\epsilon = 1.027$, $p = 0.820$, $f = 0.897$ and $\eta = 1.637$. For pure light water $L^2 = 8.1$ cm^2 and $L_s^2 = 27$ cm^2. Calculate the infinite multiplication factor, k_∞, the migration length squared, M^2, and the material buckling, B_M^2, for the reactor core. If the reactor core is an infinite slab, of thickness a in x-direction and infinite in the y- and z-directions, solve the reactor equation in the coordinate system shown. Calculate the value of a required for the reactor to be critical (i) when $\phi = 0$ on the core boundary, (ii) when $\phi = 0$ on the extrapolated boundary and (iii) when the exact boundary condition, that the

outward neutron flux vanishes at the boundary, is imposed. Assume diffusion coefficient in the core $D = 0.16$ cm. In part (ii) derive the expression for the extrapolation distance that you use. [Hint: In all cases assume that the solution for ϕ is an even function of x]

7. The core of a nuclear reactor is in the shape of a cube of side 3 m and contains 90×10^3 kg of uranium, containing 3 per cent ^{235}U, which is uniformly distributed throughout the volume of the core. Solve the reactor equation for the neutron flux, assuming that the extrapolation distance can be neglected. If the thermal power generated by the reactor is 3000 MW what is the value of the neutron flux at the centre of the core? At the centre what is the volumetric rate of heat generation $H_{\max}$? The fission cross-section of ^{235}U for thermal neutrons is 579 barns and the energy released by fission of one nucleus of ^{235}U is 200 MeV. Avogadro number is 6.023×10^{26} molecules/kmol, 1 barn $= 10^{-24}$ cm^2 and 1 eV $= 1.602 \times 10^{-19}$ J. Atomic masses of ^{235}U and ^{238}U are 235.04 and 238.05.

8. The core of a nuclear reactor has the shape of a half cylinder. In cylindrical coordinates (r, θ, z) the core occupies the region $0 \leqslant r \leqslant r_0$, $0 \leqslant \theta \leqslant \pi$, $0 \leqslant z \leqslant \ell$. Solve the reactor equation and determine the condition for the reactor to be critical. The reactor equation in cylindrical coordinates is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} + B^2 \phi = 0$$

Assume a solution of the form

$$\phi(r, \theta, z) = R(r)\Theta(\theta)Z(z)$$

Further assume that the extrapolation distance can be neglected so that ϕ can be assumed to vanish on the boundary of the core. The solution of Bessel's equation

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x\frac{\mathrm{d}y}{\mathrm{d}x} + (x^2 - \nu^2)y = 0$$

is given by

$$y = AJ_\nu(x) + BY_\nu(x)$$

where J_ν and Y_ν are the Bessel functions of the first and second kind, of order ν. Further $|Y_\nu(x)| \to \infty$ as $x \to 0$. The zeros of $J_\nu(x)$, denoted by λ_ν, satisfy $J_\nu(\lambda_\nu) = 0$. The lowest zeros for some values of ν are $\lambda_0 = 2.405$, $\lambda_1 = 3.832$, $\lambda_2 = 5.136$, $\lambda_4 = 7.588$, where we have neglected the zeros at $x = 0$ for $\nu \geqslant 1$.

What is the ratio of radius to length for which the reactor core has minimum volume? Obtain an expression for the minimum volume in terms of B.

9. The neutron flux in a thermal reactor is governed by the equation

$$\frac{1}{Dv_{\mathrm{av}}}\frac{\partial \phi_{\mathrm{th}}}{\partial t} = \nabla^2 \phi_{\mathrm{th}} + B_{\mathrm{m}}^2 \phi_{\mathrm{th}}$$

A reactor core is in the form of a cube of side a. Assuming a separable solution of the form

$$\phi_{\mathrm{th}} = X(x)Y(y)Z(z)T(t)$$

and that $\phi_{\mathrm{th}} = 0$ on the surface of the cube, show that the neutron flux is given by

$$\phi_{\mathrm{th}} = C \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{p\pi z}{a}\right) e^{(B_{\mathrm{m}}^2 - B_{\mathrm{g}}^2) D v_{\mathrm{av}} t}$$

where

$$B_{\mathrm{g}}^2 = (m^2 + n^2 + p^2)\frac{\pi^2}{a^2}$$

m, n and p are integers and C is a constant. The origin is at one corner of the cube. What would be the problem if you design your reactor so that the solution with $m = 2$, $n = p = 1$ satisfies the condition for steady state $B_{\mathrm{g}}^2 = B_{\mathrm{m}}^2$? [Hint: What would happen to a flux distribution with $m = n = p = 1$? The value of B_{g}^2 is determined not only by the physical dimension of the core but also by the values of m, n and p. We may write $(B_{\mathrm{g}}^2)_{m,n,p} = (m^2 + n^2 + p^2)\pi^2/a^2$.]

When we design the reactor so that the solution with $m = n = p = 1$ satisfies the condition for steady state, we assume that, once steady state is attained, the flux has the form

$$\sin\frac{\pi x}{a} \sin\frac{\pi y}{a} \sin\frac{\pi z}{a}$$

The initial flux could be arbitrary and can be expressed as a linear combination of solutions of the form

$$\sin\frac{m\pi x}{a} \sin\frac{n\pi y}{a} \sin\frac{p\pi z}{a}$$

Show that all modes other than the mode with $m = n = p = 1$ decay with time. For a certain reactor

$$B_{\mathrm{m}}^2 = 0.84 \times 10^{-4}\ \mathrm{cm}^{-2}, \quad D = 0.84\ \mathrm{cm}, \quad v_{\mathrm{av}} = 2.2 \times 10^5\ \mathrm{cm/s}$$

Roughly how much time would it take for a solution with $m = 2$, $n = p = 1$ to decay by a factor of $e^{-10} \approx 1/22000$?