You may use the data:
1 barn $= 10^{-24}$ cm^2, Avogadro number $N_0 = 6.023 \times 10^{23}$ mole^{-1},
1 eV $= 1.602 \times 10^{-19}$ J

1. In the sun energy is released by the nuclear reaction

$$4\ {}_1^1\text{H} \to\ {}_2^4\text{He} + 2\ {}_1^0\beta + 2\nu$$

the atomic mass of ${}_1^1$H is 1.007825 amu and of ${}_2^4$He is 4.00260 amu. The mass of ${}_1^0\beta$ or ${}_{-1}^{0}\beta$ is 0.00055 amu while the mass of the neutrino ν can be neglected. Given that the mass of the sun is 1.99×10^{30} kg, if we assume that in the beginning this entire mass is ${}_1^1$H which gets completely converted to ${}_2^4$He what is the total energy available from the nuclear reaction? Assume speed of light is 3×10^8 m/s. If the solar constant (solar energy received per unit area normal to the sun per unit time at the mean distance of the earth from the sun) is 1353 W/m^2 and the distance between the earth and the sun is 1.5×10^{11} m estimate the total power radiated by the sun. If this power is assumed to remain constant, for how many years can the fusion reaction described above provide the required energy? If we assume that there are no nuclear reactions and energy is obtained by combustion of hydrogen with oxygen which are assumed to be present in stoichiometric ratio (with total mass equal to mass of the sun) for how many years can the combustion process provide energy to sustain the present rate of energy radiated by the sun? Assume that the molecular weight of water is 18 and the energy released in combustion is 242000 kJ/kmol.

2. A nuclear reactor produces 500 MW of electrical power and has a net thermal efficiency of 33.6%. The fuel used is natural uranium (containing 0.715 percent ^{235}U) in metallic form. If all energy is assumed to come from fission of ^{235}U present in the fuel what is the rate of fuel consumed in kg of uranium metal per day? Assume that the binding energy per nucleon for ^{235}U is 7.6 MeV while for the fission products it is 8.5 MeV. Assume that the 95% of the energy released in fission is recoverable (converted to thermal energy). Use the data 1 eV $= 1.602 \times 10^{-19}$ J, Avogadro number $N_0 = 6.023 \times 10^{26}$ atoms/kmol. Atomic masses of ^{235}U and ^{238}U are 235.04 and 238.05.

   A conventional steam power plant which uses coal as fuel has an overall thermal efficiency of 38%. The calorific value of coal used is 23.1 MJ/kg. For a plant which generates 500 MW of electrical power, what is the weight of coal consumed per day?

3. A free electron decays into a proton, an electron and an anti-neutrino. In positron emission, on the other hand, a proton decays into a neutron, a positron and a neutrino. Is it possible for a free proton to decay spontaneously in this manner? The masses of elementary particles are

	Mass (u)
Proton	1.007277
Neutron	1.008665
Electron/Positron	0.00055
Neutrino/Anti-neutrino	Negligible

   In positron emission, the proton in the nucleus undergoing decay has energy less than that of a free proton by ΔE_p and the neutron produced by the decay occupies an energy level which has energy less than that of a free neutron by ΔE_n. What is the relation between ΔE_p and ΔE_n for positron decay to be energetically possible? Express your answer in terms of MeV. Use the relation 1 u $\equiv$ 931 MeV.

4. Determine the kinetic energy at which the wavelength of a neutron is comparable to (a) the diameter of a nucleus, (b) an atomic diameter, (c) the interatomic spacing in graphite, and (d) the diameter of a nuclear reactor core. (Only rough estimates are required.) Are relativistic effects important?

5. In the slowing down of neutrons by scattering collisions we use a quantity denoted by ξ. What is the definition of ξ? It can be shown that

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$$

In graphite on the average how many collisions are required to slow down a neutron from 2 MeV to 0.025 eV?

6. A neutron collides head on with a nucleus of mass number (atomic mass) A. Assuming an elastic collision, what is the fractional decrease in energy of the neutron in the laboratory frame of reference? Why are substances containing low atomic weight elements used as moderators? If scattering is isotropic in the centre of mass frame of reference it can be shown that the average decrease of log energy is given by

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right)$$

If graphite ($A = 12$) is used as the moderator what is the expected number of collisions required to slow down a neutron from 2 MeV to 0.025 eV? If no moderator is used and the same slowing down is to be obtained by collision with uranium nuclei ($A = 238$) what is the expected number of collisions required?

7. (a) Light water, H_2O, has a density 1 gm/cm^3 and molecular weight 18. Avogadro number $N_A = 6.023 \times 10^{23}$ molecules/gram mole. For a neutron the scattering and absorption cross-sections of a molecule of light water are $\sigma_s = 50$ barns and $\sigma_a = 0.66$ barns. For a neutron in light water what is the scattering mean free path (mean distance travelled between scattering collisions), the absorption mean free path (mean distance travelled up to the point where they are absorbed) and the total mean free path (mean distance travelled before any interaction, either scattering collision or absorption)? 1 barn $= 10^{-24}$ cm^2.

   (b) In a nuclear reactor the neutrons have a distribution of velocities. Let n_0 be the number density of neutrons with all possible speeds and $n(v)dv$ be the number density of neutrons with speed between v and $v + dv$. Assume that n(v) is given by

$$n(v) = C_1 v e^{-C_2 v}$$

   where C_1 and C_2 are constants. If $n(v)$ assumes its maximum value at $v = v_{\max}$, evaluate C_1 and C_2 in terms of n_0 and $v_{\max}$. Substituting these in the expression for $n(v)$ and assuming that $\Sigma(v)$ is the macroscopic cross-section for a certain type of interaction, what is the rate of such interactions per unit volume? Here $\Sigma(v)$ denotes the value of Σ for neutrons with speed v. Obtain explicit expressions for the rate of interactions per unit volume for

      i. absorption for which $\Sigma_a(v) = \Sigma_{a0}\dfrac{v_0}{v}$, where Σ_{a0} and v_0 are constants,
      ii. scattering for which $\Sigma_s(v) = \Sigma_{s0}$, where Σ_{s0} is a constant.

8. (a) In a purely scattering medium, in the continuous slowing down model, the neutron slowing-down density, q, is the rate at which neutrons slow down past any energy per unit volume. The macroscopic scattering cross-section of the medium is Σ_s. If we assume that all scattering collisions result in reduction of log (base e) of energy of the neutron by ξ then show that

$$\phi(E) = \frac{q}{\Sigma_s \xi E}$$

   where $\phi(E)dE$ is the flux of neutrons with energy between E and $E + dE$. If instead we assume that 25% of collisions result in decrease of log (base e) of energy of the neutron by 0.5ξ, 50% in decrease by ξ and the remaining 25% in decrease by 1.5ξ, what is the expression for $\phi(E)$ in terms of q?

   (b) Neutrons undergo slowing down by elastic scattering in an infinite mass of pure heavy water, D_2O. For D_2O we have $\xi = 0.509$, $\sigma_s = 10.6$ barns and $\sigma_a = 0.001$ barns. Assume that the cross-sections have these constant values for all values of energy. In the continuous slowing down model the neutron slowing-down density, $q(E)$, is the rate at which neutrons slow down past energy E per unit volume. What is the fractional reduction in q in slowing down from fission energy, 2 MeV, to thermal energy, 0.025 eV? Assume that all scattering collisions result in reduction of log (base e) of energy of the neutron by ξ.

9. The infinite multiplication factor is given by

$$k_\infty = \epsilon p f \eta$$

What is the physical meaning of each of the four factors on the right hand side of this equation. In a nuclear reactor the fuel is enriched uranium (containing 5 per cent ^{235}U) and the moderator is graphite, with 400 atoms of graphite for every atom of uranium. Calculate the value of k_∞. For fission of ^{235}U with thermal neutrons $\nu = 2.42$. For thermal neutrons:

$^{238}\mathrm{U}: \sigma_c = 2.72$ barns, $\sigma_f = 0$ barns
$^{235}\mathrm{U}: \sigma_c = 101$ barns, $\sigma_f = 579$ barns
Graphite : $\sigma_a = 0.0045$ barns

The scattering cross-sections are

Uranium: 8.3 barns
Graphite: 4.7 barns

The mean ξ for the core is approximately that for graphite which is 0.158. The resonance escape probability is given by the empirical relation

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N(^{238}U)}\right\}^{-0.514}\right]$$

where $\Sigma_s/N(^{238}U)$ is in barns.

10. In a light water reactor it is proposed to use as fuel a mixture of uranium dioxide, UO_2, and plutonium dioxide, PuO_2. Assume that the mixture contains 5 per cent PuO_2, the plutonium is pure ^{239}Pu and the uranium is natural uranium which contains 0.715 per cent ^{235}U. The percentages are in terms of number of moles/atoms. What is η for this fuel? Assume that the moderator to fuel ratio is 50 molecules of H_2O for each molecule of oxide fuel. What is the value of the infinite multiplication factor, k_∞? The resonance escape probability is given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N(^{238}U)}\right\}^{-0.514}\right]$$

where $\Sigma_s/N(^{238}U)$ is in barns and the fast fission factor is given by

$$\epsilon = \frac{1 + 0.690\left\{\frac{N(^{238}U)}{N(H_2O)}\right\}}{1 + 0.563\left\{\frac{N(^{238}U)}{N(H_2O)}\right\}}$$

Use the data, applicable for thermal neutrons:

Nuclide	σ_c (barns)	σ_f (barns)	ν	σ_s (barns)
^{238}U	2.72	0	0	8.3
^{235}U	101	579	2.42	10.0
^{239}Pu	270	745	2.91	9.7

Substance	σ_a (barns)	σ_s (barns)
O	0	3.8
H_2O	0.66	50

The mean ξ for the core can be assumed to be approximately that for H_2O which is 0.920.

11. A nuclear reactor uses natural uranium in the form of uranium dioxide UO_2 as fuel. The moderator used is heavy water D_2O while the coolant is light water H_2O. The reactor core contains 300 molecules of D_2O and 0.3 molecules of H_2O for every molecule of UO_2. Assuming a homogeneous reactor model calculate the value of the infinite multiplication factor k_∞. The resonance escape probability is given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N\left({}^{238}U\right)}\right\}^{-0.514}\right]$$

where $\Sigma_s/N({}^{238}U)$ is in barns. Assume that the fast fission factor $\epsilon \approx 1$. For fission of ^{235}U with thermal neutrons $\nu = 2.42$. For thermal neutrons:

^{238}U: $\sigma_c = 2.72$, $\sigma_f = 0$
^{235}U: $\sigma_c = 101$, $\sigma_f = 579$
O: $\sigma_a = 0$
D_2O: $\sigma_a = 0.001$
H_2O: $\sigma_a = 0.66$

The scattering cross-sections and values of ξ are:

U: $\sigma_s = 8.3$, $\xi = 0.0084$
O: $\sigma_s = 3.8$, $\xi = 0.1209$
D_2O: $\sigma_s = 10.6$, $\xi = 0.509$
H_2O: $\sigma_s = 50$, $\xi = 0.920$

All cross-sections are in barns. Natural uranium contains 0.715 percent ^{235}U (molal ratio).

12. A light water reactor is to be designed to use a mixture of uranium dioxide UO_2, containing natural uranium, and plutonium dioxide PuO_2, in which the plutonium is ^{239}Pu. In natural uranium the number of atoms of ^{235}U to the total number of U atoms is in the ratio 0.00715:1. In the mixed oxide fuel the number of molecules of PuO_2 to the number of molecules of UO_2 is 1:49. The moderator to fuel ratio is 3:1, ie, three molecules of H_2O for every molecule of fuel oxide. Calculate the infinite multiplication factor k_∞. Use the data

Substance	σ_a (barns)	σ_s (barns)	ξ
^{235}U	680	10	0.0085
^{238}U	2.72	8.3	0.0084
^{239}Pu	1015	9.7	0.0083
O	0	3.8	0.1209
H_2O	0.66	50	0.920

Substance	σ_f (barns)	ν
^{235}U	579	2.42
^{239}Pu	745	2.93

The resonance escape probability and the fast fission factor are given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N({}^{238}\mathrm{U})}\right\}^{-0.514}\right]$$

$$\epsilon = \left[1 + 0.690\frac{N({}^{238}\mathrm{U})}{N(\mathrm{H_2O})}\right] \Big/ \left[1 + 0.563\frac{N({}^{238}\mathrm{U})}{N(\mathrm{H_2O})}\right]$$

where $\Sigma_s/N({}^{238}\mathrm{U})$ is in barns.

13. Define the terms (i) thermal utilization factor, f, and (ii) eta, η. In a nuclear reactor core the fuel is a mixture of uranium dioxide, UO_2 and plutonium dioxide, PuO_2, with 9 molecules of UO_2 to 1 molecule of PuO_2. The uranium is natural uranium containing 0.715 percent ^{235}U, while the plutonium is pure ^{239}Pu. The moderator is light water, H_2O, and the reactor core contains 10 molecules of

H_2O for each molecule of oxide fuel. Calculate f and η for the reactor core. Use the data, applicable for thermal neutrons:

Nuclide	σ_c (barns)	σ_f (barns)	ν
^{238}U	2.72	0	0
^{235}U	101	579	2.42
^{239}Pu	270	745	2.91

Substance	σ_a (barns)
O	0
H_2O	0.66

14. The resonance escape probability is given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_{\mathrm{s}}}{N(^{238}\mathrm{U})}\right\}^{-0.514}\right]$$

In a nuclear reactor the fuel is uranium dioxide, UO_2. The uranium is enriched and contains 3 percent ^{235}U. Calculate p for two situations, (i) when the moderator is graphite and the moderator to fuel ratio is 200:1 and (ii) when the moderator is light water, H_2O and the moderator to fuel ratio is 5:1. When we say the moderator to fuel ratio is $n:1$ it means that there are n molecules of moderator to 1 molecule of fuel. Use the data, applicable for thermal neutrons

Substance	σ_s (barns)	ξ
^{238}U	8.3	0.00838
^{235}U	10	0.00849
O	3.8	0.1209
Graphite	4.7	0.158
H_2O	50	0.920

15. (a) A radioactive isotope has a decay constant λ. Prove that its half-life is given by

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

(b) In the present day a sustained chain reaction cannot be obtained, even for an infinite system, using natural uranium as fuel and light water as moderator. However, two billion (2×10^9) years ago such a natural reactor had operated in a uranium mine in Gabon, Africa. This was possible because the composition of natural uranium at that time was not the same as it is today. Natural uranium today contains 0.715 per cent ^{235}U. Both ^{235}U and ^{238}U undergo radioactive decay with half-lives of 7.1×10^8 years and 4.51×10^9 years, respectively. What was the percentage of ^{235}U in natural uranium two billion years ago?

(c) At present most of the nuclear reactors in India use natural uranium, in the form of uranium dioxide, UO_2, as fuel and heavy water, D_2O, as moderator. A representative of a certain country argues that India will have to switch to reactors using enriched uranium because the percentage of ^{235}U in natural uranium will keep decreasing with time and beyond a certain point it will not be possible to get a sustained chain reaction even with heavy water as moderator. To check this compute the infinite multiplication factor, k_∞, assuming UO_2 (with natural uranium) as fuel and D_2O as moderator. Assume that the moderator to fuel ratio is 400:1, i.e., 400 molecules of D_2O for each molecule of UO_2. Calculate the value of k_∞ today and 4.5×10^8 years later. Assuming a linear relation between k_∞ and time, when will we have $k_\infty = 1$? Use the data

Substance	σ_a (barns)	σ_s (barns)	ξ
^{235}U	680	10	0.0085
^{238}U	2.72	8.3	0.0084
O	0	3.8	0.1209
D_2O	0.001	10.6	0.509

Substance	σ_f (barns)	ν
^{235}U	579	2.42

The resonance escape probability is given by

$$p = \exp\left[-\frac{2.73}{\bar{\xi}}\left\{\frac{\Sigma_s}{N(^{238}\mathrm{U})}\right\}^{-0.514}\right]$$

where $\Sigma_s/N(^{238}\mathrm{U})$ is in barns. Assume that $\bar{\xi}$ can be approximated by the value for the pure moderator. Further assume that the fast fission factor $\epsilon = 1$.