# ME60223 Nuclear Power Generation & Safety

Class Test, Date: 25-10-2018, Time: 1.5 hrs, Full marks: 40

1. The core of a nuclear reactor is a cylinder of radius $R$ and length $H$. The reactor equation in cylindrical coordinates, assuming axisymmetry, is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + B^2\phi = 0$$

Using the method of separation of variables, ~~solve for the~~ neutron flux in the reactor core. Assume that the extrapolation distance can be neglected. The solution of Bessel's equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)y = 0$$

which remains finite at $x = 0$ is $J_\nu(x)$, the Bessel function of the first kind of order $\nu$. The Bessel function of order zero, $J_0(x)$, decreases from 1 at $x = 0$ to 0 at $x \approx 2.405$, the first zero of $J_0(x)$. If the reactor core contains uranium dioxide, $UO_2$, as fuel, where the uranium is natural uranium containing 0.715 percent $^{235}U$ and heavy water, $D_2O$ as moderator, with 300 molecules of $D_2O$ for every molecule of $UO_2$, calculate the dimensions of the core for the reactor to be critical, assuming $2R = H$. For the composition of the core chosen above, the four factors in the expression for $k_\infty$ have the value $\eta = 1.3247$, $f = 0.9618$, $p = 0.9187$ and $\epsilon = 1$. Use the modified one-group equation. For heavy water the square of the diffusion length $L^2 = 3\times\ 10^4$ cm$^2$ and the square of the slowing down length $L^2 = 131$ cm$^2$. Calculate the volume averaged value of $\phi$ using

$$\int_0^R J_0\left(\frac{2.405r}{R}\right) rdr = \frac{R^2}{2.405}J_1(2.405)$$

$$J_1(2.405) = 0.5190$$

15 marks

2. In reactor calculations we use the relation

$$1 - f = \frac{\Sigma_{aM}}{\bar{\Sigma}_{ac}}$$

where $\bar{\Sigma}_{ac}$ is the macroscopic absorption cross-section of the reactor core, assuming a homogeneous reactor model, and $\Sigma_{aM}$ is the macroscopic absorption cross-section of the pure moderator and $f$ is the thermal utilization factor. This is used to calculate $\bar{\Sigma}_{ac}$ if $\Sigma_{aM}$ and $f$ are known. However, if we do the derivation a little more carefully we find

$$1 - f = \frac{N_{Mc}\sigma_{aM}}{N_{Mc}\sigma_{aM} + N_{Fc}\sigma_{aF}}$$

where $\sigma_{aM}$ and $\sigma_{aF}$ are the microscopic absorption cross-sections of the moderator and of the fuel and $N_{Mc}$ and $N_{Fc}$ are the number densities of the moderator and of the fuel in gthe reactor core. We have

$$\bar{\Sigma}_{ac} = N_{Mc}\sigma_{aM} + N_{Fc}\sigma_{aF}$$

while

$$\Sigma_{Mpure} = N_{Mpure}\sigma_{aM}$$

where $\Sigma_{Mpure}$ is the macroscopic absorption cross-section of the pure moderator and $N_{Mpure}$ is the number density of the pure modderator. Then

$$\begin{aligned}\frac{\Sigma_{Mpure}}{\Sigma_{ae}} &= \frac{N_{Mpure}\sigma_{aM}}{N_{Me}\sigma_{aM} + N_{Fe}\sigma_{aF}} \\ &= \frac{N_{Me}\sigma_{aM}}{N_{Me}\sigma_{aM} + N_{Fe}\sigma_{aF}} \times \frac{N_{Mpure}}{N_{Me}} \qquad (1)\end{aligned}$$

or

$$\frac{\Sigma_{Mpure}}{\Sigma_{ae}} = (1-f) \times \frac{N_{Mpure}}{N_{Me}}$$

comparing this with the expression given in the beginning we observe that there is a correction factor $N_{Mpure}/N_{Me}$ and $N_{Me}$ is not the same as $N_{Mpure}$ since in the reactor core part of the volume is occupied by the fuel. Compute the value of this correction factor for a reactor core which contains uranium dioxide, $UO_2$, and water, $H_2O$, with 3 molecules of $H_2O$ for every molecule of $UO_2$. The densities of $H_2O$ and $UO_2$ are $10^3$ $m^3$ and $10^4$ $m^3$ and the molecular masses are 18 and 269.9

5 marks

3. In a predominantly scattering medium the partial neutron current densities in the negative and positive $x$-direction are given by

$$J_{x^-} = \frac{\phi}{4} + \frac{1}{6\Sigma_s}\frac{\partial\phi}{\partial x}$$

$$J_{x^+} = \frac{\phi}{4} - \frac{1}{6\Sigma_s}\frac{\partial\phi}{\partial x}$$

From these it follows that the neutron current density in the $x$-direction is

$$J_x = -D\frac{\partial\phi}{\partial x}$$

where

$$D = \frac{1}{3\Sigma_s}$$

using these derive the boundary condition at a plane boundary between a diffusing medium and space. Explain the concept of extrapolation distance and derive an expression for the extrapolation distance. When can the extrapolation distance be neglected?

10 marks