

1. In a nuclear reactor the reactor poison samarium,  $^{149}\text{Sm}$ , is formed by the decay of promethium,  $^{149}\text{Pm}$ , which may be considered to be a primary product of fission. The concentrations  $P$  of promethium and  $S$  of samarium are governed by the equations

$$\frac{dP}{dt} = \gamma_P \Sigma_f \phi_{th} - \lambda_P P$$

$$\frac{dS}{dt} = \lambda_P P - \sigma_{aS} \phi_{th} S$$

where  $\Sigma_f$  and  $\phi_{th}$  are the macroscopic fission cross-section and the thermal neutron flux in the reactor core,  $\gamma_P$  is the fission yield of promethium,  $\lambda_P$  is the decay constant of promethium and  $\sigma_{aS}$  is the microscopic absorption cross-section of samarium. A reactor is started up at  $t = 0$  and then operated with a constant neutron flux so that

$$\phi_{th} = 0 \quad \text{for } t < 0$$

$$\phi_{th} = 10^{14} \text{ cm}^{-2} \text{ sec}^{-1} \quad \text{for } t \geq 0$$

Assume that  $P = S = 0$  at  $t = 0$ . Derive the expressions for  $P$  and  $S$  for  $t > 0$ . Show that as  $t \rightarrow \infty$ ,  $P$  and  $S$  tend towards their equilibrium values  $P_\infty$  and  $S_\infty$ . Calculate the value of  $t$  when  $P$  reaches 90% of its equilibrium value. At that time what is the value of  $S$  compared to its equilibrium value? Use the data:  $\lambda_P = 0.0131 \text{ hr}^{-1}$ ,  $\sigma_{aS} = 5.87 \times 10^4 \text{ barns}$ ,  $1 \text{ barn} = 10^{-24} \text{ cm}^2$ .

20 marks

2. In a pressurized water reactor (PWR) the fuel consists of cylindrical pellets of uranium dioxide,  $\text{UO}_2$ , of diameter 10 mm, surrounded by a Zircaloy cladding of thickness 0.5 mm. The uranium in  $\text{UO}_2$  is 3 percent enriched. The reactor is to be designed in such a manner that the maximum temperature in the fuel at the centre of the core is  $2100^\circ\text{C}$ . The temperature of the coolant at the centre of the core is  $350^\circ\text{C}$ . What should be the rate of heat release per unit volume in the fuel and the neutron flux, at the centre of the core, for this condition to be satisfied? The thermal conductivity of  $\text{UO}_2$  is  $2.5 \text{ W/mK}$ , of Zircaloy is  $22 \text{ W/mK}$  and the heat transfer coefficient between the cladding surface and the coolant is  $20 \text{ kW/m}^2\text{K}$ . Density of  $\text{UO}_2$  is  $10 \text{ g/cm}^3$ . Atomic masses of  $^{238}\text{U}$ ,  $^{235}\text{U}$  and  $\text{O}$  are 238, 235 and 16. Avogadro's number is  $6.023 \times 10^{23} \text{ mol}^{-1}$ ,  $\sigma_f(^{235}\text{U}) = 579 \text{ barns}$  and  $1 \text{ barn} = 10^{-24} \text{ cm}^2$ . Energy released by fission of one nucleus of  $^{235}\text{U}$  is  $200 \text{ MeV}$  and  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ .

25 marks

3. A pressurized heavy water reactor (PHWR) is fuelled with uranium dioxide,  $\text{UO}_2$ , in which the uranium is natural uranium containing 0.715 percent  $^{235}\text{U}$ . The reactor is to be designed to operate for 2 years at which point it will be refuelled. If the reactor operates with a neutron flux of  $2 \times 10^{13} \text{ cm}^{-2} \text{ sec}^{-1}$ , what is the number of atoms of  $^{235}\text{U}$  and the number of atoms of  $^{239}\text{Pu}$  after 2 years ( $2 \times 365 \text{ days}$ ) of operation, per atom of uranium present at startup. The reactor uses heavy water,  $\text{D}_2\text{O}$ , as moderator, with 200 molecules of  $\text{D}_2\text{O}$  for every molecule of  $\text{UO}_2$  at startup. Calculate the value of  $k_\infty$  at startup and after 2 years of operation. In your calculations express the

number of atoms or molecules of the different substances present in the reactor core in terms of the number of atoms of uranium present at startup. Use the data

Substance	$\sigma_a$ (barns)	$\sigma_s$ (barns)	$\xi$
$^{235}\text{U}$	680	10	0.0085
$^{238}\text{U}$	2.72	8.3	0.0084
$^{239}\text{Pu}$	1008	9.6	0.0083
O	0	3.8	0.1209
$\text{D}_2\text{O}$	0.001	10.6	0.509

Substance	$\sigma_f$ (barns)	$\nu$
$^{235}\text{U}$	579	2.42
$^{239}\text{Pu}$	742	2.93

The resonance escape probability is given by

$$p = \exp \left[ -\frac{2.73}{\xi} \left\{ \frac{\Sigma_a}{N(^{238}\text{U})} \right\}^{-0.514} \right]$$

where  $\Sigma_a/N(^{238}\text{U})$  is in barns. Since the moderator to fuel ratio is very large the values of  $\Sigma_a$  and  $\xi$  can be taken to be those for the pure moderator. The fast fission factor  $\epsilon$  can be taken to be unity.  $1 \text{ barn} = 10^{-24} \text{ cm}^2$ .

25 marks

4. The core of a nuclear reactor has the shape of a cube of side  $a$ . Solve the reactor equation for the neutron flux in the reactor core

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + B^2 \phi = 0$$

using the method of separation of variables. Assume that the extrapolation distance can be neglected. What should be the value of  $a$  for the reactor to be critical? The reactor core contains uranium dioxide,  $\text{UO}_2$ , with 3 percent enriched uranium as fuel, and water,  $\text{H}_2\text{O}$ , as moderator. There are 3 molecules of  $\text{H}_2\text{O}$  for every molecule of  $\text{UO}_2$ . For this combination of fuel and moderator, the four factors in the expression for  $k_\infty$  have the values  $\eta = 1.8246$ ,  $f = 0.9209$ ,  $p = 0.7930$  and  $\epsilon = 1.0348$ . For water,  $\text{H}_2\text{O}$ , the square of diffusion length  $L^2 = 8.1 \text{ cm}^2$  and the square of slowing down length  $L_s^2 = 27 \text{ cm}^2$ . Using the modified one group equation compute the value of  $a$  for which the reactor will be critical.

If the thermal power of the reactor is 10 MW what is the average neutron flux in the reactor core? What is the neutron flux at the centre of the core? The energy released by fission of one nucleus of  $^{235}\text{U}$  is 200 MeV and  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ . For the fuel moderator combination chosen, in the reactor core  $N(^{235}\text{U}) = 2.231 \times 10^{20} \text{ cm}^{-3}$ . The microscopic fission cross-section  $\sigma_f(^{235}\text{U}) = 579 \text{ barns}$  and  $1 \text{ barn} = 10^{-24} \text{ cm}^2$ .

25 marks

5. (a) What is the out-in scheme for refuelling nuclear reactors and why is it used?  
 (b) Explain what is the safety risk if a nuclear reactor is over moderated.

5 marks