introduce irregularities in the thermal neutron distribution, that would not be present if the reactor were homogeneous, throughout the pile.

The effect on the slow neutron flux plotted along a line passing right through the core (but not passing through the fuel elements) has also been determined experimentally for BEPO, and the curve is given in Fig. 6.7.

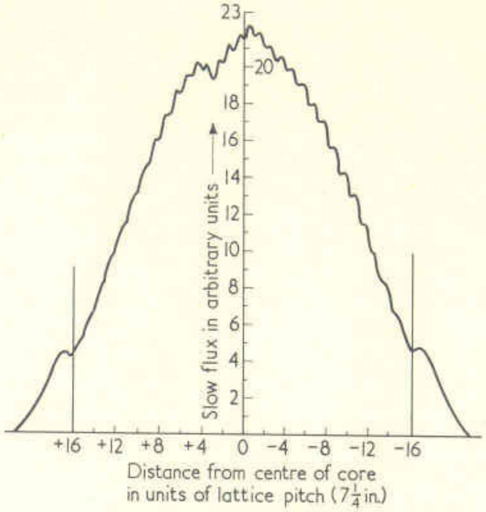

Fig. 6.7. Slow neutron flux distribution across core of BEPO. Reprinted with permission from Fenning, *Progress in Nuclear Energy*, Series II, Vol. 1. London; Pergamon Press.

When the distribution of slow neutron flux is known for a lattice cell, it is merely a matter of integration to calculate the ratio of the average flux in the moderator to the average flux in the uranium, this being greater than unity. The average thermal neutron flux in the uranium determines the rate of fission; the average flux in the moderator is directly related to the absorption of neutrons in the moderator and structural materials in the core, and to their escape from the core. The excess flux in the moderator due to the lumping of the fuel therefore increases neutron wastage. The ratio of mean flux of thermal neutrons in the moderator to mean flux of thermal neutrons in the uranium is known as the disadvantage factor for thermal neutrons, or thermal disadvantage

factor: it has to do with neutrons, not with heat. The disadvantage is one associated with the lumping of the fuel in a heterogeneous reactor.

Fast Fission Factor

We have discussed the fuel element as a sink for slow neutrons. The absorption of the slow neutrons causes fission, as a result of which fast neutrons are emitted, and the fuel element becomes a source of fast neutrons. There is a small chance, though not an unimportant one, that a fast neutron from a slow fission will make a collision before escaping from the fuel element, and cause a fast fission. If this occurs, more than one fast neutron is emitted, and there is a net gain of fast neutrons. The ratio of fast neutrons escaping from the fuel lump to the number liberated by slow fission is called the fast fission factor ε. The fast fission factor represents an advantage of the heterogeneous reactor compared with the homogeneous.

We now consider the neutron economy in the fuel element in so far as the fast neutrons are concerned, following the life history of n fast neutrons originating in slow fission. If p is the probability that one of these neutrons makes some sort of collision before escaping from the uranium, then $n(1 - p)$ first generation neutrons escape without collision.

Of the np neutrons that make collisions, $np\sigma_f/\sigma$ bring about second generation fission, where σ_f is the cross-section of natural uranium for fast fission, and σ the cross-section for collisions of any kind. The number of (fast) neutrons emitted per fission in natural uranium being ν, there are $np\nu\sigma_f/\sigma$ fast fission neutrons contributed to the second generation for n in the first. There will also be $np\sigma_e/\sigma$ neutrons from elastic collisions, where σ_e is the corresponding cross-section. Now the most probable energy for neutrons emitted in fission is about 1 MeV, and the threshold for fast fission in ^{238}U has nearly the same value. We shall therefore assume, as an approximation, that any fission neutron that has had an inelastic collision is no longer capable of causing fast fission. There are $np\sigma_i/\sigma$ such neutrons, where σ_i is the cross-section for inelastic collisions. Neutrons that suffer capture without fission are lost. The second generation therefore consists of

$$\frac{np\nu\sigma_f}{\sigma} + \frac{np\sigma_e}{\sigma} = np\left(\frac{\nu\sigma_f + \sigma_e}{\sigma}\right) = npZ,$$

say, neutrons having sufficient energy to cause further fission, and $np\sigma_i/\sigma$ which have been slowed down below the fast fission threshold. The probability of second generation neutrons making a collision before escaping from the fuel lump is not necessarily quite the same as for the first generation, but, as an approximation we shall assume it to be so. The error incurred will not be great, since the series we are going to sum converges fairly quickly.

Of the npZ neutrons capable of causing further fission, $np(1-p)Z$ will escape, and np^2Z remain to start a third generation. Most of the $np\sigma_i/\sigma$ neutrons will escape, because the absorption cross-section is small below the fission threshold, and elastic and inelastic collisions will not eliminate them, though slowing them a little. The total number of escaping second generation neutrons is, therefore, $np(1-p)Z + np\sigma_i/\sigma$, whereas it was $n(1-p)$ for the first. The second generation was started by np neutrons; we have seen that there are np^2Z to start the third. In general, successive generations are started by pZ times as many neutrons. The total number of escaping neutrons is therefore

$n(1-p)$	from the first generation
$+\, np(1-p)Z + np\,\sigma_i/\sigma$	from the second generation
$+\, np^2(1-p)Z^2 + np^2Z\,\sigma_i/\sigma$	from the third generation
$+\, np^3(1-p)Z^3 + np^3Z^2\,\sigma_i/\sigma$	from the fourth generation
$+$ etc.	

Rearranging, we have

$$n(1-p+p\,\sigma_i/\sigma)(1+pZ+p^2Z^2+p^3Z^3+\ldots)$$
$$= \frac{n(1-p+p\,\sigma_i/\sigma)}{1-pZ}$$
$$= n\varepsilon$$

since the fast fission factor ε is defined as the factor by which the number of fast neutrons emitted in slow fission is multiplied

to give the number of (fast) neutrons which escape from the fuel lump. This gives

$$\varepsilon = 1 + \frac{p\,\dfrac{\sigma_f}{\sigma}\left\{(\nu-1) - \dfrac{\sigma_c}{\sigma_f}\right\}}{1 - p\left\{\dfrac{\nu\sigma_f + \sigma_e}{\sigma}\right\}}$$

since $\sigma = \sigma_f + \sigma_i + \sigma_e + \sigma_c$ where σ_c is the cross-section for capture (without fission).

Table 6.1

Fast neutron cross-sections in barns, for natural uranium.*

Elastic scattering	1·5
Inelastic scattering	2·47
Radiative capture	0·04
Fission	
$\nu = 2·55$	0·29

Using the data * given in Table 6.1 we find

$$\varepsilon = 1 + \frac{0{\cdot}095\,p}{1 - 0{\cdot}52\,p}$$

for natural uranium. $\varepsilon - 1$ is plotted as a function of p in Fig. 6.8.

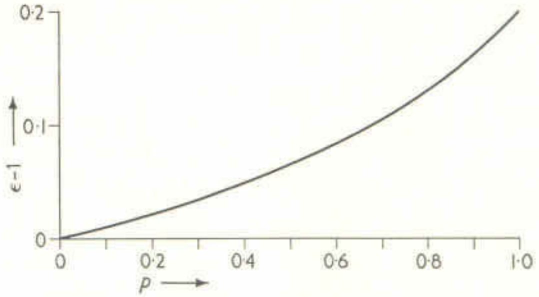

Fig. 6.8. Dependence of fast fission factor on probability of collision before escape from natural uranium fuel element.

* These are not the most recent figures, but continue to be used because they combine with the approximate theory to give good results. *cf.* page 304.

Since p cannot be greater than 1, the maximum value of ε is 1·20. $Z = (\nu\sigma_f + \sigma_e)/\sigma$ takes the value 0·52 for natural uranium, and this is also the maximum value of pZ. Since successive generations of neutrons are started by pZ times as many neutrons, it follows that a lump of natural uranium fuel cannot become self-sustaining: for the chain-reaction to be divergent it would be necessary for $pZ > 1$.

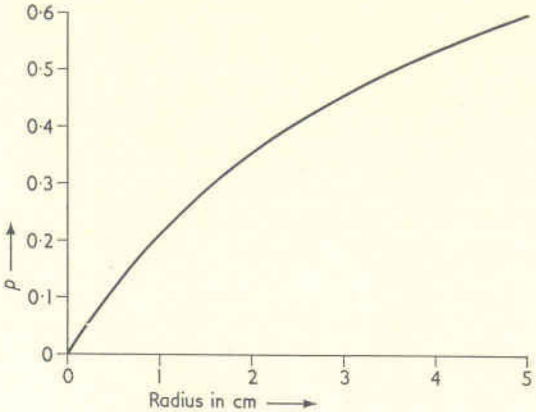

Fig. 6.9. Probability of collision before escape from rod-shaped fuel element, for a mean free path of 5·0 cm.

The probability p of a fission neutron making a collision in a fuel lump before escaping depends on the mean free path and the geometry of the lump. It can be computed for any particular case by integration. The values for cylindrical rods of different diameters are given in Fig. 6.9*.

For the fuel rods in BEPO which are 0·9 in. in diameter, we get $\varepsilon = 1{\cdot}03$. In practice, then, the gain in number of neutrons from fast fission in the fuel elements is only a few per cent.

* Littler, D. J. and Raffle, J. F. *An Introduction to Reactor Physics*, p. 115, 1957. London; Pergamon Press.

CHAPTER 7

The Fast Reactor

Fission neutrons liberated in the fuel rods of a natural uranium pile cause some further neutrons to be produced by fast fission in uranium-238 before they leave the rods, but, as we have just seen, the increase in the number of neutrons amounts to only a few per cent in practice, and could not, even in principle, exceed 20 per cent. This limitation comes from the existence of a fission threshold in ^{238}U, and the fact that neutrons easily lose enough energy in inelastic collisions to fall below the fission threshold. In a lump of uranium-235, on the other hand, there is no fission threshold, and neutrons can still provoke fission after having suffered inelastic collisions, with the result that their number can multiply greatly, even to the extent that a homogeneous mass of uranium-235 can become critical as a result of fast fission alone. We shall, therefore, reconsider the treatment of fast neutron economy used to calculate the fast fission factor for uranium-238, now considering the case of uranium-235. In this case attention is focused on the neutrons that stay behind rather than on those that escape from the fuel.

Fast Neutron Economy in Uranium-235

We start with n neutrons originating in fast fission, and again let p represent the average probability that one of these neutrons makes some sort of a collision before escaping from the uranium. Of the np collisions, $np\sigma_a/\sigma$ result in absorption with or without fission, where σ_a is the cross-section for absorption, and σ the cross-section for collisions of any kind. We have $np\sigma_s/\sigma$ collisions resulting in scattering, with or without loss of energy, but we shall not now need to distinguish between these two alternatives, though it would be important to do so if we were designing a fast reactor, and to allow for the dependence of cross-sections on neutron energy. σ_s is the cross-section for scattering, and $\sigma = \sigma_a + \sigma_s$. At the first stage, then, there are $np\sigma_a/\sigma$ neutrons absorbed, and $np\sigma_s/\sigma$ left to make further collisions. As in the treatment