theory be applied to fast neutrons in a fuel rod, since the mean free path for fast neutrons is about twice that for slow.

Random Movement

The track of a particle (such as a neutron), if it consists of straight sections joined together by sharp bends, looks something like Fig. 6.2. It is apparent that such a particle, starting at A in a particular direction, makes very poor progress in that direction: after a while it may be found almost anywhere, though it is likely to have moved some distance from A. Its rate of progress in such a case will be very much less than that indicated by its track speed. We shall now show that this progress is proportional to the square root of the time. If the track speed is v, there will have been $vt/\bar{\lambda}$ collisions after time t, where $\bar{\lambda}$ is the arithmetic average length of path between collisions. This is the same as the mean free path λ of the last section, but we now need to be precise, because we shall shortly meet another average. We shall be concerned only with the case of $vt/\bar{\lambda}$ large, so that statistical considerations can be fairly applied.

We begin by considering a one-dimensional movement, in which the particle moves backwards and forwards along the x axis. Let successive displacements be $x_1, x_2 \ldots, x_n$, the resultant displacement X being $x_1 + x_2 + \ldots + x_n$. The resultant displacement squared is $(x_1 + x_2 + \ldots + x_n)^2$. Expanding, this becomes

$$(x_1^2 + x_2^2 + \ldots + x_n^2) + 2\sum\sum x_p x_q$$

in which the double sum * is taken over all values of p and q from 1 to n excluding $p = q$. Now let us average the expression over a large number of occasions when n successive displacements occur, in order to find $\overline{X^2}$. It is clear that $\overline{x_1^2} = \overline{x_2^2} = \ldots = \overline{x_n^2} = \overline{x^2}$, say. The first term in the expansion of $\overline{X^2}$ is therefore $n\overline{x^2}$. When taking the average of the double sum, a given positive and the same negative displacement are equally likely to be among the

* Σ, which mathematicians usually reserve for a summation sign, is used in the literature of nuclear engineering also to represent a macroscopic collision 'cross-section'.

values of x_p and x_q, and will cancel out. The double sum is therefore zero, and $\overline{X^2} = n\overline{x^2}$.

We now consider tracks in three dimensions, and assume first that the collisions are isotropic in their scattering effect, all scattering directions being equally probable, so that, after scattering, the particle has no 'memory' of its initial direction of motion. Successive paths $\lambda_1, \lambda_2 \ldots, \lambda_n$ are completely independent. A free path λ_p is really a vector with components $\lambda_{p,x}, \lambda_{p,y}, \lambda_{p,z}$, and $\lambda_p^2 = \lambda_{p,x}^2 + \lambda_{p,y}^2 + \lambda_{p,z}^2$. The resultant displacement $\mathbf{R}$ is a vector whose components are

$$R_x = \lambda_{1,x} + \lambda_{2,x} + \ldots + \lambda_{n,x}$$
$$R_y = \lambda_{1,y} + \lambda_{2,y} + \ldots + \lambda_{n,y}$$
$$R_z = \lambda_{1,z} + \lambda_{2,z} + \ldots + \lambda_{n,z}$$

and $\mathbf{R}^2 = R_x^2 + R_y^2 + R_z^2$.

Now $R_x^2 = (\lambda_{1,x}^2 + \lambda_{2,x}^2 + \ldots + \lambda_{n,x}^2) + 2\sum\sum \lambda_{p,x}\lambda_{q,x}$ and the double sum will vanish when averaged, for the same reason as before. Thus $\overline{R_x^2} = n\overline{\lambda_x^2}$, say. Therefore

$$\overline{\mathbf{R}^2} = n(\overline{\lambda_x^2} + \overline{\lambda_y^2} + \overline{\lambda_z^2})$$

Since summation and averaging are commutative,

$$\overline{\mathbf{R}^2} = n(\overline{\lambda_x^2 + \lambda_y^2 + \lambda_z^2})$$
$$= n\overline{\lambda^2}$$

Taking square roots, and remembering that $n = vt/\bar{\lambda}$, we see that the root mean square distance travelled is proportional to the root of the time, and equal to the root mean square free path times the root of the number of free paths in that time. The significant free path is not $\bar{\lambda}$, but $(\overline{\lambda^2})^{\frac{1}{2}}$. Recalling that the probability of a collision between x and $x + \delta x$ is $\lambda^{-1} e^{-x/\lambda}\delta x$, where λ has the significance of $\bar{\lambda}$, we see that

$$\overline{\lambda^2} = \frac{\int_0^\infty x^2(\bar{\lambda})^{-1} e^{-x/\lambda}\, dx}{\int_0^\infty (\bar{\lambda})^{-1} e^{-x/\lambda}\, dx}$$

Integrating by parts gives $\overline{\lambda^2} = 2\bar{\lambda}^2$.

We therefore find that the mean square distance travelled is $\overline{\mathbf{R}^2} = 2n\bar{\lambda}^2$, in which $n = vt/\bar{\lambda}$.