operation of a reactor with a graphite moderator it is necessary to carry out a controlled release of Wigner energy at intervals. The release is potentially the more dangerous the lower the working temperature of the reactor, and must be carried out according to a carefully thought out schedule.

An alternative procedure proposed depends on the fact that the major part of the Wigner energy is stored in those parts of the graphite nearest the fuel rods. If, then, the fuel rods are surrounded by graphite tubes, and these are removed from the reactor with the rods when fuel reprocessing is due, the dangerous graphite can be annealed outside the reactor or replaced by fresh graphite.

Transport Mean Free Path

The amount of moderator needed round each fuel element in a heterogeneous reactor depends on the distance travelled by a neutron while being slowed down. We have already calculated the number of collisions involved, and the mean free path between them is the inverse of the macroscopic scattering cross-section, but this only tells us the distance travelled measured along the zig-zag path, and we require the direct distance between start and finish. We obtained a formula for this on page 103, but must now generalize it in terms of the transport mean free path.

In calculating the root mean square distance travelled by a particle executing random motion, we assumed random collisions terminating the free paths, and isotropic scattering in the laboratory system of reference at collisions, whereas we now know it to be isotropic in the centre of mass system for the case we are interested in. This means that, instead of all evidence of the direction of motion before collision being lost in each collision, there is a tendency for the initial motion to persist in the subsequent motion: there is a preponderance of forward scattering. The mean free path that is effective in making a particle wander away from its starting point is longer than λ_s. We shall assume that its value may be calculated by allowing for the contribution to continued motion in the direction of the free path that was terminated by the collision from free paths following the collision. The value for the transport mean free path obtained in this way is the same as

that obtained from a formal mathematical proof along the lines of the one we used in Chapter 6 for isotropic scattering.*

We simplify the argument by treating all free paths as equal to the mean free path λ_s. Consider a particle starting at O in Fig. 8.9 and travelling along the X axis. It collides after a distance λ_s, and sets off at an inclination θ to its former direction, again colliding after a distance λ_s. The contribution from the second free path, to motion in the direction of the first, is $\lambda_s \cos\theta$, and the average contribution from all such particles is $\lambda_s \overline{\cos\theta}$. The second collision occurs at a distance λ_s from the vertex of a cone of semi-vertical angle θ, on the surface of the cone, the axis of the cone being the direction of motion before the first collision, and

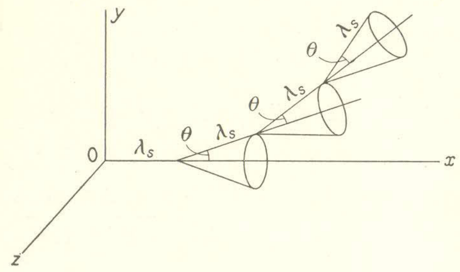

Fig. 8.9.

the vertex at the collision. A similar construction gives the third collision. The contribution from the third free path to the transport mean free path is got by first projecting on to the axis of the second cone, and then projecting on to the axis of the first. It is therefore $\lambda_s (\overline{\cos\theta})^2$. The contribution from the fourth free path is $\lambda_s (\overline{\cos\theta})^3$, and so on. We therefore find

$$\lambda_{tr} = \lambda_s\{1 + \overline{\cos\theta} + (\overline{\cos\theta})^2 + (\overline{\cos\theta})^3 + \ldots\}$$

$$= \frac{\lambda_s}{1 - \overline{\cos\theta}}$$

* Littler, D. J. and Raffle, J. F. *An Introduction to Reactor Physics*, Appendix I. 1957, London; Pergamon.

$\overline{\cos\theta}$ must be calculated from the fact that scattering is isotropic in the centre of mass system, that is, referring to Fig. 8.2, all directions in space are equally probable for $u = Av_0/(1 + A)$. As we have noted previously, this means that the probability of the scattering angle in the centre of mass system lying between ϕ and $\phi + \delta\phi$ is $\frac{1}{2}\sin\phi\delta\phi$. Again referring to Fig. 8.2, we see that

$$\tan\theta = \frac{\dfrac{Av_0}{1+A}\sin\phi}{\dfrac{Av_0}{1+A}\cos\phi + \dfrac{v_0}{1+A}}$$

$$= \frac{A\sin\phi}{A\cos\phi + 1}$$

Now

$$\sec^2\theta = 1 + \tan^2\theta$$

$$= 1 + \frac{A^2\sin^2\phi}{(A\cos\phi + 1)^2}$$

$$= \frac{A^2 + 1 + 2A\cos\phi}{(A\cos\phi + 1)^2}$$

whence

$$\cos\theta = \frac{A\cos\phi + 1}{(A^2 + 1 + 2A\cos\phi)^{\frac{1}{2}}}$$

We therefore have

$$\overline{\cos\theta} = \tfrac{1}{2}\int_0^\pi \frac{A\cos\phi + 1}{(A^2 + 1 + 2A\cos\phi)^{\frac{1}{2}}}\sin\phi\, d\phi$$

$$= \tfrac{1}{2}\int_{-1}^{+1} \frac{Ax + 1}{(A^2 + 2Ax + 1)^{\frac{1}{2}}}\, dx.$$

The integration is straightforward, and gives $\overline{\cos\theta} = 2/(3A)$, whence $\lambda_{tr} = \lambda_s/[1 - 2/(3A)]$.

This is the correct value of λ_{tr} to use in the formula for the diffusion coefficient derived on page 104, $D = \frac{1}{3}v\lambda_{tr}$. We have to be careful, however, about using it in the formula for the root mean square distance travelled derived on page 103, $\overline{\mathbf{R}^2} = 2n\bar{\lambda}^2$. It is incorrect to substitute λ_{tr}^2 for $\bar{\lambda}^2$. This is because one $\bar{\lambda}$ in the formula has to do with the rate of progress per collision, and the other with movement along the zig-zag path; if we differentiate the formula with respect to time, one $\bar{\lambda}$ has to do with the *track velocity*. Explicitly, if n is the number of collisions in time τ, we have $n = v\tau/\bar{\lambda}$, whence $\overline{\mathbf{R}^2} = 2v\bar{\lambda}\tau = 6D\tau$, since $D = \frac{1}{3}v\bar{\lambda}$. In generalizing, we must use λ_s for $\bar{\lambda}$ in $n = v\tau/\bar{\lambda}$, and λ_{tr} for $\bar{\lambda}$ in $D = \frac{1}{3}v\bar{\lambda}$. The generalization of $\overline{\mathbf{R}^2} = 2n\bar{\lambda}^2$ will therefore be $\overline{\mathbf{R}^2} = 2n\lambda_s\lambda_{tr}$, in order to retain $\overline{\mathbf{R}^2} = 6D\tau$.

We can now define the slowing down length L_s, in terms of the root mean square distance travelled between the birth of a fission neutron and its thermalization. We let $L_s^2 = \frac{1}{6}\overline{\mathbf{R}^2} = \frac{1}{3}n_{th}\lambda_s\lambda_{tr} = D\tau$. It is convenient to have no factor 6 present when L_s is associated with D. n_{th} is now the number of collisions to thermalize, which we calculated.

Slowing Down Formulae

The conclusions reached in the elementary theory of the moderator presented in this chapter will now be summarized.

The energy of a neutron after collision with a moderator atom of atomic weight A is

$$E_0\frac{A^2 + 2A\cos\phi + 1}{(A + 1)^2}$$

where E_0 is the energy before collision, and ϕ the angle through which the neutron is scattered in the centre of mass system (θ being the angle through which it is scattered in the laboratory system). The minimum value of E is

$$\alpha E_0 = E_0\left(\frac{A - 1}{A + 1}\right)^2$$

The probability of the neutron being in a range of energy δE, within the range αE_0 to E_0 which includes all the neutrons, is independent of the energy after collision and equal to $\delta E/\{E_0(1 - \alpha)\}$, as shown in Fig. 8.3.

The lethargy is defined as $\mathscr{L} = \log_e(E_f/E)$ where E_f is the energy of a fission neutron. The average increase of lethargy